



PROBLEM 5. Constants,  $r$ , Indep. var  $\theta$ ,  $0 \leq \theta \leq \frac{\pi}{2}$

Dep. vars,  $x = r \cos \theta$ ,  $y = r \sin \theta$ ,  $h = 2y = 2r \sin \theta$ ,

$$V = \pi (x)^2 \cdot h = 2\pi r^3 (\cos \theta)^2 \sin \theta = 2\pi r^3 (1 - |\sin \theta|^2) \sin \theta$$

To maximize,  $V(\theta) = 2\pi r^3 (1 - |\sin \theta|^2) \sin \theta$ ,  $V(0) = 0$ ,  $V(\frac{\pi}{2}) = 0$

Critical points,  $V'(\theta) = 2\pi r^3 (1 - 3(\sin \theta)^2) \cdot \cos \theta$ ;  $\theta = \frac{\pi}{2}$ ,  $\theta = \arcsin(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$

$$V(\frac{\pi}{6}) = 2\pi r^3 \left(\frac{\sqrt{3}}{\sqrt{3}}\right)^2 \cdot \frac{1}{\sqrt{3}} = \frac{4}{3} \pi r^3 \cdot \left(\frac{1}{\sqrt{3}}\right)$$