MAT131 Fall 2010 Midterm 2

Name: _____ SB ID number: _____

Please circle the number of your recitation.

1. TuTh 12:50 – SB Union Gianniotis

3. TuTh 8:20 – SB Union Lee

4. WF 11:45 – Lgt Engr Lab Boyd

5. MF 2:20 – Library Kim

6. MW 10:40 – Physics Medina

7. MW 6:50 – Physics Atyam

8. MW 3:50 – Old Chem Medina

9. TuTh 5:20 – SB Union Gianniotis

11. TuTh 3:50 – SB Union Lee

12. TuTh 8:20 – SB Union Wroten

13. MF 12:50 – Physics Kim

********** **DO NOT WRITE BELOW THIS LINE.** **********

Problem 1: _____ /10	**Problem 3**: _____ /20	**Problem 5**: _____ /20
Problem 2: _____ /20	**Problem 4**: _____ /15	**Problem 6**: _____ /15
		TOTAL: _____ /100

Instructions: The exam is closed book, closed notes, calculators are not allowed, and all cell phones and other electronic devices must be turned off for the duration of the exam. You will have approximately 90 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., then **please raise your hand**.

Name: _____ **Problem 1**: _____ /10

Problem 1(10 points) In each of the following statements, circle T if it is true and F if it is false. Each part is worth **only 2 points out of 100 total points**. Remember to use your time wisely. There is no need to show your work on this problem.

T F (a) If differentiable functions $f(x)$ and $g(x)$ are both 0 at $x = a$, then also the derivative of $f(x)g(x)$ equals 0 at $x = a$.

T F (b) The only everywhere differentiable functions which are equal to their own fourth derivatives are $f(x) = \sin(x)$ and $f(x) = \cos(x)$.

T F (c) The derivative of every inverse trigonometric function is another inverse trig function.

T F (d) For differentiable functions $f(x)$ and $g(x)$, if $g'(a)$ equals 0 then the derivative at $x = a$ of $f(g(x))$ equals 0.

T F (e) For every real number $a > 0$, the derivative of $f(x) = a^x$ is an everywhere positive function.

Name: _____ **Problem 2**: _____ /20

Problem 2(20 points) In each of the following cases, compute the derivative. You may use the derivative formulas for $\sin(x)$, $\cos(x)$, e^x and $\ln(x)$, and the derivative rules such as the product rule, quotient rule and chain rule. But you must show all other work.

Show your work and write a box or circle around your final answer.

(a)(4 points)

$$\frac{d}{dx}\sqrt{1-\cos^2(x)}, \quad \text{for } 0 < x < \pi/2$$

(b)(6 points)

$$\frac{d}{dx}x^{\ln(x)}$$

Hint. Use logarithms.

Name: _____ **Problem 2, continued**

In each of the following cases, compute the derivative. **Show all your work and write a box or circle around your final answer.**

(c)(6 points)

$$\frac{d}{dx}\left(\frac{x-x^{-1}}{x+x^{-1}}\right).$$

(d)(4 points)

$$\frac{d}{dx}\cos(\arctan(x)).$$

Name: _____ **Problem 3**: _____ /20

Problem 3(20 points) A playground slide has the shape of a diagonal line whose top is a point 6 feet directly above a plaque on the ground and whose bottom is a point on the ground 8 feet horizontally distant from the plaque. A child on the slide moves away from the top at a speed of 1/2 feet per second at the moment when she is halfway down the slide. At that moment, how fast is the distance from the child to the plaque increasing?

Show all your work, including a labeled diagram and a list of equations relating the various quantities of the problem. Write your answer as a fraction, a/b feet per second. Write a box or circle around your final answer.

Name: _____ **Problem 4**: _____ /15

Problem 4(15 points) The inverse function of $\tan(x)$ is $\tan^{-1}(x) = \arctan(x)$. Derive the formula for the derivative of $\arctan(x)$.
You are allowed to use the formulas you know for the derivatives of $\sin(x)$, $\cos(x)$ or $\tan(x)$. You may also use the trigonometric formula relating $\sin^2(x)$ and $\cos^2(x)$.

Show all of the steps in your derivation. If a step is missing or is not clearly written, you will lose points.

Name: _____ **Problem 5**: _____ /20

Problem 5(20 points) Consider the smooth curve in the plane with implicit equation

$$y^2 = x^3 - 3x - 1.$$

(a)(10 points) Find the equation of the tangent line to the curve at the point $(2, 1)$.
Show all your work. Write your answer in slope-intercept form $y = mx + b$.

$$\boldsymbol{y} = \_\_\_\_\_$$

(b)(10 points) Find **both** the x- and y-coordinates of all points on the curve where the tangent line has slope 0. **Show all your work. Write each point as $(x, y) = (a, b)$ where a and b are numbers.**

Name: _____ **Problem 6**: _____ /15

Problem 6(15 points) Find a whole number whose fourth power is close to 15. Use this to find the approximate value of $(15)^{3/4}$ using a linear approximation or differentials.

Show all your work. Write your approximate answer as a proper fraction $a\frac{b}{c}$.

$(15)^{3/4}$ approximately equals _____

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