MAT131 Fall 2007 Final Exam

Name: _____ SB ID number: _____

Please circle the number of your recitation.

| 1. TuTh 12:50 – Library | 9. TuTh 5:20 – Library | | |
|-------------------------|----------------------------------|------------|------|
| Rajaguru | Cernea | Problem 1: | /50 |
| 2. MF 2:20 – Library | 10. WF $9:35 - \text{Library}$ | | |
| Ionas | Koca | Problem 2: | /40 |
| 3. TuTh 8:20 – Library | 11. TuTh $3:50 - \text{Library}$ | | |
| Balsam | Cernea | Problem 3: | /30 |
| 4. WF 11:45 – Library | 12. TuTh $8:20 - Library$ | | |
| Ionas | Girao | Problem 4: | /25 |
| 5. TuTh 2:20 – Library | 13. MF 12:50 – Library | | |
| Rajaguru | Koca | Problem 5: | /25 |
| 6. MW $10:40 - Physics$ | 14. TuTh 3:50 – Physics | | |
| Balsam | Girao | Problem 6: | /10 |
| 7. MW $6:50 - Library$ | | | |
| Mirzaei | | Problem 7: | /20 |
| 8. MW 3:50 – Library | | | |
| Ionas | | Total: | /200 |
| | | , | |

Instructions: Please write your name at the top of the front page of your exam booklet. The exam is closed book, closed notes, calculators are not allowed, and all cellphones and other electronic devices must be turned off for the duration of the exam. You will have approximately 2.5 hours for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Problem 1(50 points) For the function $y = x^{-1} \ln(e \cdot x^2)$ do each of the following. Show all your work. In Parts (a), (b), (c) and (d), write a box or circle around your final answers.]

(a)(2 points) Say whether this is an even function, an odd function, or neither an even nor odd function.

(b)(10 points) Find all vertical and horizontal asymptotes.

Name: _____

Name: ______

Problem 1, continued

(c)(10 points) Find all critical points and say where the function is increasing and where it is decreasing.

(d)(10 points) Find all inflection points and say where the function is concave up and where it is concave down.

Problem 1, continued

(e)(18 points) Sketch the graph of y = f(x). Label all asymptotes, all local maxima and minima and all inflection points.

Problem 2: _____ /40

Problem 2(40 points) Let P be a point in the first quadrant on the unit circle and let Q be the reflection of P through the *y*-axis. A quadrilateral has vertices (1,0), P, Q and (-1,0). For which point P is the area of the quadrilateral maximal? Show all your work and write a box or circle around your final answer.

Name: _____

Name: _____

Problem 3(30 points) Compute each of the following indefinite and definite integrals. Show all your work and write a box or circle around your final answers.

(a)(2 points)

$$\int \frac{x^2 + 1}{x} dx$$

(b)(4 points)

$$\int \frac{x}{x^2 + 1} dx$$

(c)(3 points)

$$\int \frac{1}{x^2 + 1} dx$$

Name:

Problem 3, continued

(d)(6 points)

$$\int \frac{\sin(2\theta)}{1+\cos^2(\theta)} d\theta.$$

(e)(5 points)

$$\int_0^{\pi/4} \frac{\sin(\theta)}{1 - \sin^2(\theta)} d\theta$$

(f)(3 points)

$$\int_{1}^{4} \left(\sqrt{y} - \frac{1}{\sqrt{y}}\right) dy$$

Name:

Problem 3, continued

(g)(1 point)

$$\int_{1}^{e^{0}} \frac{2x\ln(\sqrt{x^{2}+1})}{x^{2}+1} dx$$

(h)(6 points)

$$\int_0^1 \frac{2x\ln(\sqrt{x^2+1})}{x^2+1} dx$$

Problem 4: _____ /25

Problem 4(25 points) A particle moves to the right along the line y = 3 meters. At the moment when the distance from the origin to the particle equals 5 meters, this distance is increasing at the instantaneous rate of 8 meters per second. Find the speed of the particle at this moment. Show all your work and write a box or circle around your final answer.

Name: _____

Name: _____

Problem 5(25 points) Compute each of the following limits. Show all your work and write a box or circle around your final answers.

(a)(4 points)

 $\lim_{x\to 0} \frac{\arcsin(x)}{x}$

(b)(4 points)

 $\lim_{x\to\infty} x^2 e^{1/x^2}$

(c)(3 points)

$$\lim_{x \to 3^+} \frac{x^2 - 2x + 3}{\sqrt{x^2 - 9}}$$

Name: _____

Problem 5, continued

(d)(4 points)

 $\lim_{x \to \infty} ((x^2 + 1)^{1/2} - x)$

(e)(10 points)

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n+2i}$$

Name:

Problem 6(10 points) For $f(x) = x^3$ compute f'(2) as a limit of a difference quotient. (Evaluate the limit directly, do not evaluate using L'Hospital's rule.) Show all your work and write a box or circle around your final answers.

Problem 7(20 points) In each of the following cases, compute the derivative. Show all your work and write a box or circle around your final answers.

(a)(4 points)

 $y = x^{\ln(x)} \qquad \qquad y' = ?$

(b)(4 points)

$$y = \arcsin(\sqrt{1 - x^2}) \qquad \qquad y' = ?$$

(c)(4 points) Find y' at the point $(x, y) = (5\sqrt{3}/4, 5/4)$ where y satisfies the implicit equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ Name: ______

Problem 7, continued

(d)(4 points)

$$y = \int_0^{\sqrt{x}} e^{-t^2} dt \qquad \qquad y' = ?$$

(e)(4 points)

$$y = \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + x}}}$$
 $y' =?$