# MAT 123 Practice for Core Competency Exam B With Solutions

**Remark.** If you are comfortable with all of the following problems, you will be well prepared for Core Competency Exam B. For the Core Competency Exams, passing will be 85% or better.

**Exam Policies.** You must show up on time for all exams. Please bring your student ID card: ID cards may be checked, and students may be asked to sign a picture sheet when turning in exams. Other policies for exams will be announced at the beginning of the exam.

If you have a university-approved reason for taking an exam at a time different than the scheduled exam (because of a religious observance, a student-athlete event, etc.), please contact your instructor as soon as possible. Similarly, if you have a documented medical emergency which prevents you from showing up for an exam, again contact your instructor as soon as possible.

For excused absences from a Core Competency Exam, since there will be multiple attempts for such an exam, usually the student will simply be asked to pass one of the later attempts. In exceptional circumstances, a make-up exam may be scheduled for the missed exam.

All exams are closed notes and closed book. Once the exam has begun, having notes or books on the desk or in view will be considered cheating and will be referred to the Academic Judiciary.

For all exams, you must bring your Stony Brook ID. The IDs may be checked against picture sheets.

It is not permitted to use cell phones, calculators, laptops, MP3 players, Blackberries or other such electronic devices at any time during exams. If you use a hearing aid or other such device, you should make your instructor aware of this before the exam begins. You must turn off your cell phone, etc., prior to the beginning of the exam. If you need to leave the exam room for any reason before the end of the exam, it is still not permitted to use such devices. Once the exam has begun, use of such devices or having such devices in view will be considered cheating and will be referred to the Academic Judiciary. Similarly, once the exam has begun any communication with a person other than the instructor or proctor will be considered cheating and will be referred to the Academic Judiciary.

**Review Topics.**
The following are the core skills for the second part of the course.

(1) Understand the graph of a parabola, a circle, an ellipse, and a hyperbola. Understand how these differ from one another.

(2) Be able to simplify expressions involving polynomials, including sums, differences, multiplication and factoring one polynomial into another (without remainder).

(3) Simplify expressions involving radicals and exponents, including fractional exponents.

(4) Solve equations involving exponents.

(5) Recognize simple powers and radicals, such as $3^4$ and $32^{1/5}$.

(6) Recognize the graphs of exponential growth and exponential decay.

(7) Recognize the graphs of logarithm functions with various bases.

(8) Simplify expressions involving logarithms.

(9) Solve equations involving logarithms.

## Practice Problems.

**(1)** Please look at the six conic sections at the following website.

`http://www.purplemath.com/modules/conics.htm+`.

For each one, please identify it as parabola, circle, ellipse or hyperbola.

**Solution to (1)** Reading from left to right and then top to bottom, these are
parabola, circle, hyperbola, parabola, ellipse, hyperbola.

**(2)** In each of the following cases, identify the simplified expression.

(i) $1 + 2^1 + 2^2 + 2^3$

$$(a)\ 2^4, \quad (b)\ 14, \quad (c)\ 2^4 - 1, \quad (d)\ 17.$$

(ii) $(27)^{2/3}$

$$(a)\ 82\sqrt{3}, \quad (b)\ 9, \quad (c)\ \sqrt[3]{726}, \quad (d)\ 18.$$

(iii) $10^{\sqrt{4}}$

$$(a)\ 100, \quad (b)\ \sqrt[4]{10}, \quad (c)\ 10000, \quad (d)\ 20.$$

(iv) $5^{6/5}/5^{-4/5}$

$$(a)\ 5^{2/5}, \quad (b)\ \sqrt[5]{15625}/\sqrt[5]{625}, \quad (c)\ \sqrt[5]{25}, \quad (d)\ 25.$$

(v) $1 + x^{1/3} + x^{2/3}$

$$(a)\ (x-1)/(x^{1/3}-1), \quad (b)\ 1 + 3x^{1/3}, \quad (c)\ \sqrt[3]{1 + x + x^2}, \quad (d)\ 1 + x^{-1/3}(1 + x).$$

(vi) $x^{1/3}x^{-1/2}$

$$(a)\ x^{5/6},\quad (b)\ 1/x^{1/6},\quad (c)\ x^{-5/6},\quad (d)\ (-1/6)x.$$

(vii) $(x^{1/3})^{1/2}$

$$(a)\ x^{5/6},\quad (b)\ \sqrt[6]{x^5},\quad (c)\ x^{1/6},\quad (d)\ (1/6)x.$$

(viii) $((x^2y^3/(x^{-3}y^2)^{-3})^2$

$$(a)\ x^4y^6/(x^{-9}y^6),\quad (b)\ (x^2y^3)(x^{-18}y^{12}),\quad (c)\ (y^9/x^6)^2,\quad (d)\ x^{-14}y^{18}.$$

(ix) $\sqrt[3]{(81x^6)/(125y^3)}$

$$(a)\ 3\sqrt[3]{3}x^2/(5y),\quad (b)\ 9x^3/(5y\sqrt{5y},\quad (c)\ 9x^2/(5y),\quad (d)\ 9x^2/(5y\sqrt{5}).$$

**Solution to (2)** (i) The sum $1+2^1+2^2+2^3$ equals $1+2+4+8$, which equals 15. Among the four options, this is the same as $\boxed{\text{(c)}}$, $2^4-1=16-1=15$.

(ii) By the exponent rules, $27^{1/3}$ equals the cube root $\sqrt[3]{27}$. This equals 3. Thus, by the exponent rules, $27^{2/3}$ equals $(27^{1/3})^2$, which equals $3^2$. Therefore the answer is $\boxed{\text{(b)}}$, 9.

(iii) The square root of 4 equals 2. Thus, this is $10^2$, which equals 100. Therefore the answer is $\boxed{\text{(a)}}$.

(iv) By the exponent rules, $5^{6/5}/5^{-4/5}$ equals $5^{6/5}\cdot 5^{4/5}$. By the exponent rules, this equals $5^{(6/5)+(4/5)}$. Since $6/5+4/5$ equals $10/5$, which equals 2, this gives $5^2$. Therefore $5^{6/5}/5^{-4/5}$ equals 25 so that the answer is $\boxed{\text{(d)}}$.

(v) The correct answer is $\boxed{\text{(a)}}$. To see this, clear denominators and observe that $(x^{1/3}-1)(1+x^{1/3}+x^{2/3})$ distributes to $x^{1/3}+x^{2/3}+x^{3/3}-1-x^{1/3}-x^{2/3}$. After cancellation, this gives $x-1$. None of the other answers is correct, as is verified by substituting $x=27$.

(vi) By the exponent rules, $x^{1/3}x^{-1/2}$ equals $x^{(1/3)-(1/2)}$. Since $1/3-1/2$ equals $-1/6$, $x^{1/3}x^{-1/2}$ equals $x^{-1/6}$. By the exponent rules, this is the same as $1/x^{1/6}$. Therefore the correct answer is $\boxed{\text{(b)}}$. None of the other answers is correct, as is verified by substituting $x=64=2^6$.

(vii) By the exponent rules, $(x^{1/3})^{1/2}$ equals $x^{(1/3)\cdot(1/2)}$. Since $(1/3)\cdot(1/2)$ equals $1/6$, $(x^{1/3})^{1/2}$ equals $x^{1/6}$. Thus the correct answer is $\boxed{\text{(c)}}$. None of the other answers is correct, as is verified by substituting $x=64=2^6$.

(viii) By the exponent rules, $(x^{-3}y^2)^{-3}$ equals $(x^{-3})^{-3}\cdot(y^2)^{-3}$. By the exponent rules, $(x^{-3})^{-3}$ equals $x^{(-3)(-3)}=x^9$. Similarly, $(y^2)^{-3}$ equals $y^{2(-3)}=y^{-6}$. Thus, altogether, $(x^{-3}y^2)^{-3}$ equals

$x^9y^{-6}$. So the fraction $x^2y^3/(x^{-3}y^2)^{-3}$ equals $x^2y^3/(x^9y^{-6})$. By the exponent rules, this equals $(x^2y^3)(x^{-9}y^6)$. By associativity and commutativity of multiplication, this equals $(x^2 \cdot x^{-9})(y^3 \cdot y^6)$. By the exponent rules, this equals $x^{2-9}y^{3+6}$, which equals $x^{-7}y^9$. Thus, $(x^2y^3/(x^{-3}y^2)^{-3})^2$ equals $(x^{-7}y^9)^2$. By the exponent rules, this equals $(x^{-7})^2(y^9)^2$, which in turn equals $x^{(-7)2}y^{9\cdot 2}$. Therefore, finally, $(x^2y^3/(x^{-3}y^2)^{-3})^2$ equals $x^{-14}y^{18}$. So the correct answer is $\boxed{\text{(d)}}$. None of the other answers is correct, as is verified by substituting $(x, y) = (-1, 1)$, $(x, y) = (1, -1)$ or $(x, y) = (2, 2)$.

(ix) By the exponent rules, this equals $((81x^6)/(125y^3))^{1/3}$, which in turn equals $(81)^{1/3}(x^6)^{1/3}/((125)^{1/3}(y^3)^{1/3})$. Of course 81 equals $3^4$ and 125 equals $5^3$. Thus, $(81)^{1/3}$ equals $3\sqrt[3]{3}$, and $(125)^{1/3}$ equals 5. By the exponent rules, $(x^6)^{1/3}$ equals $x^{6(1/3)}$, which in turn equals $x^2$. Similarly, $(y^3)^{1/3}$ equals $y^{3(1/3)}$, which in turn equals $y$. Therefore, the final answer is $3\sqrt[3]{3}x^3/(5y)$. So the correct answer is $\boxed{\text{(a)}}$. None of the other answers is correct, as is verified by substituting $(x, y) = (1, 1)$.

**(3)** In each of the following cases, simplify the given expression.

(i) $(1 + x + x^2)(1 - x)$

$$(a)\ 1 - x + x^2 - x^3,\quad (b)\ 1 - x + x - x^2 + x^2 + x^3,\quad (c)\ 1 - x^3,\quad (d)\ 1 - x^2 + x^2 - x^4.$$

(ii) $(x^3 + 2x^2 - 7x - 2)/(x - 2)$

$$(a)\ x^2 + 4x + 1,\quad (b)\ x^2 + 1,\quad (c)\ x^2 - 4x + 1,\quad (d)\ x + 5.$$

(iii) $(1 + x^3)^2 - (1 - x^3)^2$

$$(a)\ 2+2x^6,\quad (b)\ (1+2x^3+x^6)-(1-2x-x^6),\quad (c)\ ((1+x^3)+(1-x^3))((1+x^3)-(1-x)^3),\quad (d)\ 4x^3.$$

(iv) $f(g(x)),\ f(s) = s^2 - 1,\ g(t) = t^2 + 1$

$$(a)\ (t^2 + 1)^2 - 1,\quad (b)\ x^2(x^2 + 2),\quad (c)\ x^4 + 2x^2 + 1,\quad (d)\ (x^2 - 2)(x^2 + 2).$$

**Solution to (3)** (i) The product $(1 + x + x^2)(1 - x)$ distributes to $1 + x + x^2 - x - x^2 - x^3$. After cancellation, this equals $1 - x^3$. Thus the correct answer is $\boxed{\text{(c)}}$. None of the other answers is correct, as is verified by substituting $x = -1$.

(ii) Polynomial division of $x^3 + 2x^2 - 7x - 2$ by $x - 2$ gives $x^2 + 4x + 1$ (this also follows from synthetic division). Thus the correct answer is $\boxed{\text{(a)}}$. None of the other answers is correct, as is verified by substituting in $x = 0$ and $x = 1$.

(iii) The product $(1 + x^3)^2$ distributes to $1 + 2x^3 + x^6$. Similarly, the product $(1 - x^3)^2$ distributes to $1 - 2x^3 + x^6$. Thus the difference $(1 + x^3)^2 - (1 - x^3)^2$ equals $(1 + 2x^3 + x^6) - (1 - 2x^3 + x^6)$, which cancels to $4x^3$. Thus the correct answer is $\boxed{\text{(d)}}$. None of the other answers is correct, as is verified by substituting in $x = -1$.

(iv) The composition $f(g(x))$ equals $(g(x))^2 - 1$, which equals $(x^2+1)^2 - 1$. The product $(x^2+1)^2$ distributes to $x^4+2x^2+1$. Thus, $(x^2+1)^2-1$ equals $(x^4+2x^2+1)-1$, which cancels to $x^4+2x^2$. This factors as $x^2(x^2+2)$. Therefore the correct answer is $\boxed{\text{(b)}}$. The first answer is incorrect because it is not even a polynomial in $x$. The other answers is incorrect, as is verified by substituting $x = 0$.

**(4)** In each of the following cases, solve for the variable.

(i) $3^x = 1/27$.

$$(a)\ x = 3, \quad (b)\ x = 2, \quad (c)\ x = -3, \quad (d)\ x = 1/81.$$

(ii) $2^y = 64$.

$$(a)\ y = 4, \quad (b)\ y = 5, \quad (c)\ y = 32, \quad (d)\ y = 6.$$

(iii) $3^x = 15$.

$$(a)\ x = 15/3, \quad (b)\ x = \sqrt[3]{15}, \quad (c)\ x = \log_2(15), \quad (d)\ x = 1 + \log_3(5).$$

(iv) $\sqrt{x}^u = x^{1+x}$.

$$(a)\ u = 2 + 2x, \quad (b)\ u = 2 + x, \quad (c)\ u = \log_x(x^{2+x}), \quad (d)\ u = 1 + x.$$

(v) $2^{x^2}4^x = 8$.

$$(a)\ x = \sqrt{\log_2(8 \cdot 4^x)}, \quad (b)\ x = 1 \text{ and } x = -3, \quad (c)\ x = 1, \quad (d)\ x = -3.$$

**Solution to (4)** (i) Since 27 equals $3^3$, $1/27$ equals $3^{-3}$. Thus the equation is $3^x = 3^{-3}$. Since $f(x) = 3^x$ is a one-to-one function, the unique solution is $x = -3$. Thus the correct answer is $\boxed{\text{(c)}}$.

(ii) Since 64 equals $2^6$, the equation is $2^y = 2^6$. Since $f(y) = 2^y$ is a one-to-one function, the unique solution is $y = 6$. Thus the correct answer is $\boxed{\text{(d)}}$.

(iii) Taking the logarithm base 3 of each side of the equation gives $\log_3(3^x) = \log_3(15)$. By the definition of logarithm, $\log_3(3^x)$ equals $x$. Thus the solution is $x = \log_3(15)$. Since 15 equals $3 \cdot 5$, by the logarithm rules, $\log_3(15)$ equals $\log_3(3) + \log_3(5)$. By the definition of logarithm, $\log_3(3)$ equals 1. Thus, the solution is $x = 1 + \log_3(5)$. So the correct answer is $\boxed{\text{(d)}}$.

(iv) By definition, $x^{1/2}$ equals $\sqrt{x}$. Thus, $\sqrt{x}^u$ equals $(x^{1/2})^u$. By the exponent rules, this equals $x^{(1/2)u}$. So the equation is $x^{u/2} = x^{1+x}$. Since for positive $x$ with $x \neq 1$ the function $f(y) = x^y$ is a one-to-one function, the unique solution is $u/2 = 1 + x$. Therefore, $u$ equals $2 + 2x$. The correct answer is $\boxed{\text{(a)}}$.

(v) Since 4 equals $2^2$, also $4^x$ equals $(2^2)^x$. By the exponent rules, this equals $2^{2x}$. Thus $2^{x^2}4^x$ equals $2^{x^2}2^{2x}$. By the exponent rules, this equals $2^{x^2+2x}$. Since 8 equals $2^3$, the equation is $2^{x^2+2x} = 2^3$.

Since the function $f(x) = 2^x$ is a one-to-one function, the solution is $x^2 + 2x = 3$. Completing the square gives $x^2 + 2x + 1 = 4$, i.e., $(x + 1)^2 = 4$. The solutions are $x + 1 = \pm 2$, i.e., $x = -3$ and $x = 1$. Thus the correct answer is $\boxed{\text{(b)}}$.

**(5)** For each of the following exponential functions, say whether the function is increasing (the graph rises to the right) or decreasing (the graph rises to the left).

$$\text{(i) } f(x) = 3^x, \quad \text{(ii) } f(x) = (0.25)^x, \quad \text{(iii) } f(x) = 2^{-x}, \quad \text{(iv) } f(x) = -10 \cdot 3^x.$$

**Solution to (5)** (i) The function $f(x) = b^x$ is increasing for $b > 1$, and it is decreasing for $0 < b < 1$. Thus $3^x$ is $\boxed{\text{increasing}}$.

(ii) Since $0 < 0.25 < 1$, $(0.25)^x$ is $\boxed{\text{decreasing}}$.

(iii) By the exponent rules, $2^{-x}$ equals $(1/2)^x$. Since $0 < 1/2 < 1$, $(1/2)^x$ is $\boxed{\text{decreasing}}$.

(iv) Since $3 > 1$, the function $3^x$ is increasing. Since $-10$ is negative, $-10 \cdot 3^x$ is $\boxed{\text{decreasing}}$.

**(6)** Identify the simplified expression.

(i) $\log_2(8)$.

$$(a)\ 4, \quad (b)\ 3, \quad (c)\ 256, \quad (d)\ 3\log_2(3),$$

(ii) $\log_3(27x^2)$.

$$(a)\ \log_3(27) + (\log_3(x))^2, \quad (b)\ 3 + \log_2(x^2), \quad (c)\ 3(1 + \log_3(x)), \quad (d)\ 3 + 2\log_3(x).$$

(iii) $\log_2(4^{1+x})$.

$$(a)\ 1 + x\log_2(4), \quad (b)\ (1 - x)\log_2(4), \quad (c)\ 2 + 2x, \quad (d)\ 1 + x.$$

(iv) $\log_4(64)/\log_4(2)$.

$$(a)\ \log_2(64), \quad (b)\ 3/2, \quad (c)\ 6/(1/2), \quad (d)\ 12.$$

**Solution to (6)** (i) Since 8 equals $2^3$, by the definition of logarithm, $\log_2(8)$ equals 3. Thus the correct answer is $\boxed{\text{(b)}}$.

(ii) By the logarithm rules, $\log_3(27x^2)$ equals $\log_3(27)+\log_3(x^2)$. Since 27 equals $3^3$, by the definition of logarithm, $\log_3(27)$ equals 3. By the logarithm rules, $\log_3(x^2)$ equals $2\log_3(x)$. Thus $\log_3(27x^2)$ equals $3 + 2\log_3(x)$. Thus the correct answer is $\boxed{\text{(d)}}$.

(iii) By the logarithm rules, $\log_2(4^{1+x})$ equals $(1 + x)\log_2(4)$. Since 4 equals $2^2$, by the definition of logarithm, $\log_2(4)$ equals 2. Thus $\log_2(4^{1+x})$ equals $(1 + x)2$, which in turn equals $2 + 2x$. Thus the correct answer is $\boxed{\text{(c)}}$.

(iv) Since 64 equals $4^3$, by the definition of logarithm, $\log_4(64)$ equals 3. Since 2 equals $\sqrt{4}$, i.e., $4^{1/2}$, by the definition of logarithm, $\log_4(2)$ equals $1/2$. Thus $\log_4(64)/\log_4(2)$ equals $3/(1/2)$, which equals 6. Since $\log_2(64)$ also equals 6, the correct answer is $\boxed{\text{(a)}}$. Alternatively, this follows directly by the "change of base" rule for logarithms.

**(7)** In each of the following cases, solve for the variable.

(i) $\log_5(x-1) = 2$.

$$(a)\ x = 1 + \log_5(2),\quad (b)\ x = 5^2 \cdot 5^1,\quad (c)\ 1 + (2/\log_5),\quad (d)\ x = 26.$$

(ii) $\log_2(8x) = 5$.

$$(a)\ x = 4,\quad (b)\ x = 2^5 \cdot 8,\quad (c)\ x = 5 - \log_2(8),\quad (d)\ x = 16.$$

(iii) $\log_2(x)/\log_2(3) = 2$.

$$(a)\ x = 2\log_2(3),\quad (b)\ x = 2^2 \cdot 3,\quad (c)\ x = 9,\quad (d)\ x = 2^3.$$

(iv) $\log_2(x+1) - \log_2(x-1) = 2$.

$$(a)\ x = 1,\quad (b)\ x = -1,\quad (c)\ x = 2^2 - 1,\quad (d)\ x = 5/3.$$

**Solution to (7)** (i) Since $\log_5(x-1)$ equals 2, by the definition of logarithm, $x-1$ equals $5^2$. Thus, $x$ equals $5^2+1$, which equals 26. Thus the correct answer is $\boxed{\text{(d)}}$.

(ii) Since $\log_2(8x)$ equals 5, by the definition of logarithm, $8x$ equals $2^5$. Thus, $x$ equals $2^5/8$, which equals 4. Thus the correct answer is $\boxed{\text{(a)}}$.

(iii) By the "change of base" rule for logarithms, $\log_2(x)/\log_2(3)$ equals $\log_3(x)$. Since $\log_3(x)$ equals 2, by the definition of logarithm, $x$ equals $3^2$, which equals 9. Thus the correct answer is $\boxed{\text{(c)}}$.

(iv) By the logarithm rules, $\log_2(x+1) - \log_2(x-1)$ equals $\log_2((x+1)/(x-1))$. Since $\log_2((x+1)/(x-1))$ equals 2, by the definition of logarithm, $(x+1)/(x-1)$ equals $2^2$, which equals 4. Thus $x+1$ equals $4(x-1)$, i.e., $x+1$ equals $4x-4$. Thus $3x$ equals 5, i.e., $x$ equals $5/3$. So the correct answer is $\boxed{\text{(d)}}$.

**(8)** For each of the following logarithm functions, say whether the function is increasing (the graph is rising to the right) or decreasing (the graph is rising to the left).

$$\text{(i)}\ \log_{10}(x),\quad \text{(ii)}\ \log_2(1/x),\quad \text{(iii)}\ \log_{0.5}(x),\quad \text{(iv)}\ -10\log_3(x).$$

**Solution to (8)** (i) The function $f(x) = \log_b(x)$ is increasing for $b > 1$, and it is decreasing for $0 < b < 1$. Thus, $\log_{10}(x)$ is increasing.

(ii) By the logarithm rules, $\log_2(1/x)$ equals $-\log_2(x)$. Since $2 > 1$, $\log_2(x)$ is increasing. Since $-1$ is negative, $-\log_2(x)$ is decreasing. Alternatively, $\log_2(1/x)$ equals $\log_{0.5}(x)$, and this is decreasing since $0 < 0.5 < 1$.

(iii) As explained above, $\log_{0.5}(x)$ is decreasing.

(iv) Since $3 > 1$, $\log_3(x)$ is increasing. Since $-10$ is negative, $-10\log_3(x)$ is decreasing.