MAT 123 Practice for Core Competency Exam B

Remark. If you are comfortable with all of the following problems, you will be well prepared for Core Competency Exam B. For the Core Competency Exams, passing will be 85% or better.

Exam Policies. You must show up on time for all exams. Please bring your student ID card: ID cards may be checked, and students may be asked to sign a picture sheet when turning in exams. Other policies for exams will be announced at the beginning of the exam.

If you have a university-approved reason for taking an exam at a time different than the scheduled exam (because of a religious observance, a student-athlete event, etc.), please contact your instructor as soon as possible. Similarly, if you have a documented medical emergency which prevents you from showing up for an exam, again contact your instructor as soon as possible.

For excused absences from a Core Competency Exam, since there will be multiple attempts for such an exam, usually the student will simply be asked to pass one of the later attempts. In exceptional circumstances, a make-up exam may be scheduled for the missed exam.

All exams are closed notes and closed book. Once the exam has begun, having notes or books on the desk or in view will be considered cheating and will be referred to the Academic Judiciary.

For all exams, you must bring your Stony Brook ID. The IDs may be checked against picture sheets.

It is not permitted to use cell phones, calculators, laptops, MP3 players, Blackberries or other such electronic devices at any time during exams. If you use a hearing aid or other such device, you should make your instructor aware of this before the exam begins. You must turn off your cell phone, etc., prior to the beginning of the exam. If you need to leave the exam room for any reason before the end of the exam, it is still not permitted to use such devices. Once the exam has begun, use of such devices or having such devices in view will be considered cheating and will be referred to the Academic Judiciary. Similarly, once the exam has begun any communication with a person other than the instructor or proctor will be considered cheating and will be referred to the Academic Judiciary.

Review Topics.
The following are the core skills for the second part of the course.

(1) Understand the graph of a parabola, a circle, an ellipse, and a hyperbola. Understand how these differ from one another.

(2) Be able to simplify expressions involving polynomials, including sums, differences, multiplication and factoring one polynomial into another (without remainder).

(3) Simplify expressions involving radicals and exponents, including fractional exponents.

(4) Solve equations involving exponents.

(5) Recognize simple powers and radicals, such as 3^4 and $32^{1/5}$.

(6) Recognize the graphs of exponential growth and exponential decay.

(7) Recognize the graphs of logarithm functions with various bases.

(8) Simplify expressions involving logarithms.

(9) Solve equations involving logarithms.

Practice Problems.

(1) Please look at the six conic sections at the following website.

$$\texttt{http://www.purplemath.com/modules/conics.htm+}.$$

For each one, please identify it as parabola, circle, ellipse or hyperbola.

(2) In each of the following cases, identify the simplified expression.
(i) $1 + 2^1 + 2^2 + 2^3$

$\qquad (a)\ 2^4,\quad (b)\ 14,\quad (c)\ 2^4 - 1,\quad (d)\ 17.$

(ii) $(27)^{2/3}$

$\qquad (a)\ 82\sqrt{3},\quad (b)\ 9,\quad (c)\ \sqrt[3]{726},\quad (d)\ 18.$

(iii) $10^{\sqrt{4}}$

$\qquad (a)\ 100,\quad (b)\ \sqrt[4]{10},\quad (c)\ 10000,\quad (d)\ 20.$

(iv) $5^{6/5}/5^{-4/5}$

$\qquad (a)\ 5^{2/5},\quad (b)\ \sqrt[5]{15625}/\sqrt[5]{625},\quad (c)\ \sqrt[5]{25},\quad (d)\ 25.$

(v) $1 + x^{1/3} + x^{2/3}$

$\qquad (a)\ (x-1)/(x^{1/3}-1),\quad (b)\ 1+3x^{1/3},\quad (c)\ \sqrt[3]{1+x+x^2},\quad (d)\ 1+x^{-1/3}(1+x).$

(vi) $x^{1/3}x^{-1/2}$

$$(a)\ x^{5/6}, \quad (b)\ 1/x^{1/6}, \quad (c)\ x^{-5/6}, \quad (d)\ (-1/6)x.$$

(vii) $(x^{1/3})^{1/2}$

$$(a)\ x^{5/6}, \quad (b)\ \sqrt[6]{x^5}, \quad (c)\ x^{1/6}, \quad (d)\ (1/6)x.$$

(viii) $((x^2y^3/(x^{-3}y^2)^{-3})^2$

$$(a)\ x^4y^6/(x^{-9}y^6), \quad (b)\ (x^2y^3)(x^{-18}y^{12}), \quad (c)\ (y^9/x^6)^2, \quad (d)\ x^{-14}y^{18}.$$

(viii) $\sqrt[3]{(81x^6)/(125y^3)}$

$$(a)\ 3\sqrt[3]{3}x^2/(5y), \quad (b)\ 9x^3/(5y\sqrt{5y}), \quad (c)\ 9x^2/(5y), \quad (d)\ 9x^2/(5y\sqrt{5}).$$

(3) In each of the following cases, simplify the given expression.
(i) $(1 + x + x^2)(1 - x)$

$$(a)\ 1 - x + x^2 - x^3, \quad (b)\ 1 - x + x - x^2 + x^2 + x^3, \quad (c)\ 1 - x^3, \quad (d)\ 1 - x^2 + x^2 - x^4.$$

(ii) $(x^3 + 2x^2 - 7x - 2)/(x - 2)$

$$(a)\ x^2 + 4x + 1, \quad (b)\ x^2 + 1, \quad (c)\ x^2 - 4x + 1, \quad (d)\ x + 5.$$

(iii) $(1 + x^3)^2 - (1 - x^3)^2$

$$(a)\ 2 + 2x^6, \quad (b)\ (1 + 2x^3 + x^6) - (1 - 2x - x^6), \quad (c)\ ((1 + x^3) + (1 - x^3))((1 + x^3) - (1 - x)^3), \quad (d)\ 4x^3.$$

(iv) $f(g(x))$, $f(s) = s^2 - 1$, $g(t) = t^2 + 1$

$$(a)\ (t^2 + 1)^2 - 1, \quad (b)\ x^2(x^2 + 2), \quad (c)\ x^4 + 2x^2 + 1, \quad (d)\ (x^2 - 2)(x^2 + 2).$$

(4) In each of the following cases, solve for the variable.
(i) $3^x = 1/27$.

$$(a)\ x = 3, \quad (b)\ x = 2, \quad (c)\ x = -3, \quad (d)\ x = 1/81.$$

(ii) $2^y = 64$.

$$(a)\ y = 4, \quad (b)\ y = 5, \quad (c)\ y = 32, \quad (d)\ y = 6.$$

(iii) $3^x = 15$.

$$(a)\ x = 15/3, \quad (b)\ x = \sqrt[3]{15}, \quad (c)\ x = \log_2(15), \quad (d)\ x = 1 + \log_3(5).$$

(iv) $\sqrt{x^u} = x^{1+x}$.

$$(a)\ u = 2 + 2x, \quad (b)\ u = 2 + x, \quad (c)\ u = \log_x(x^{2+x}), \quad (d)\ u = 1 + x.$$

(v) $2^{x^2} 4^x = 8$.

$$(a)\ x = \sqrt{\log_2(8 \cdot 4^x)}, \quad (b)\ x = 1 \text{ and } x = -3, \quad (c)\ x = 1, \quad (d)\ x = -3.$$

(**5**) For each of the following exponential functions, say whether the function is increasing (the graph rises to the right) or decreasing (the graph rises to the left).

$$(i)\ f(x) = 3^x, \quad (ii)\ f(x) = (0.25)^x, \quad (iii)\ f(x) = 2^{-x}, \quad (iv)\ f(x) = -10 \cdot 3^x.$$

(**6**) Identify the simplified expression.
   (i) $\log_2(8)$.

$$(a)\ 4, \quad (b)\ 3, \quad (c)\ 256, \quad (d)\ 3\log_2(3),$$

(ii) $\log_3(27x^2)$.

$$(a)\ \log_3(27) + (\log_3(x))^2, \quad (b)\ 3 + \log_2(x^2), \quad (c)\ 3(1 + \log_3(x)), \quad (d)\ 3 + 2\log_3(x).$$

(iii) $\log_2(4^{1+x})$.

$$(a)\ 1 + x\log_2(4), \quad (b)\ (1 - x)\log_2(4), \quad (c)\ 2 + 2x, \quad (d)\ 1 + x.$$

(iv) $\log_4(64)/\log_4(2)$.

$$(a)\ \log_2(64), \quad (b)\ 3/2, \quad (c)\ 6/(1/2), \quad (d)\ 12.$$

(**7**) In each of the following cases, solve for the variable.
   (i) $\log_5(x - 1) = 2$.

$$(a)\ x = 1 + \log_5(2), \quad (b)\ x = 5^2 \cdot 5^1, \quad (c)\ 1 + (2/\log_5), \quad (d)\ x = 26.$$

(ii) $\log_2(8x) = 5$.

$$(a) \ x = 4, \quad (b) \ x = 2^5 \cdot 8, \quad (c) \ x = 5 - \log_2(8), \quad (d) \ x = 16.$$

(iii) $\log_2(x)/\log_2(3) = 2$.

$$(a) \ x = 2\log_2(3), \quad (b) \ x = 2^2 \cdot 3, \quad (c) \ x = 9, \quad (d) \ x = 2^3.$$

(iv) $\log_2(x+1) - \log_2(x-1) = 2$.

$$(a) \ x = 1, \quad (b) \ x = -1, \quad (c) \ x = 2^2 - 1, \quad (d) \ x = 5/3.$$

(**8**) For each of the following logarithm functions, say whether the function is increasing (the graph is rising to the right) or decreasing (the graph is rising to the left).

$$(i) \ \log_{10}(x), \quad (ii) \ \log_2(1/x), \quad (iii) \ \log_{0.5}(x), \quad (iv) \ -10\log_3(x).$$