Name: ______________________________ **Problem 1:** ____________ /25

Problem 1(25 points) The following table gives the **preference schedule** for an election with four candidates.

Number of voters	6	5	4
1^{st} place	A	C	D
2^{nd} place	B	B	B
3^{rd} place	C	A	A
4^{th} place	D	D	C

(a)(5 points) Which candidate wins under the **plurality-with-elimination** method (sometimes also called "instant runoff")? **Show all your work.**

Cand.	# 1^{st} place
A	6
B	0 ← Elim.
C	5
D	4

⇝ New Pref Sched

	6	5	4
1^{st}	A	C	D
2^{nd}	C	A	A
3^{rd}	D	D	C

⇝

Cand.	# 1^{st} place
A	6
C	5
D	4 ← Elim.

⇝ New Pref. Sched

	10	5
1^{st}	A	C
2^{nd}	C	A

⇝

Cand	# 1^{st} Place
A	10
C	5 ← Elim.

$\boxed{\text{A wins}}$

(b)(10 points) This election has a **Condorcet candidate**. Find the Condorcet candidate, and state whether or not the Condorcet criterion is satisfied. **Show all your work.**

A vs. B

B	9 ✓
A	6

B vs. C

B	10 ✓
C	5

B vs. D

B	11 ✓
D	4

Since B wins every pairwise comparison, B is the Condorcet candidate. Since the Condorcet candidate lost, the Condorcet ~~cand~~ criterion is violated.

(c)(10 points) Candidate C drops out of the race, but otherwise all relative rankings remain the same. Determine the new winner under plurality-with-elimination, and state which fairness criterion is violated by this outcome. **Show all your work.**

New Pref. Sched.

	6	5	4
1^{st}	A	B	D
2^{nd}	B	A	B
3^{rd}	D	D	A

⇝

Cand.	# 1^{st} Place
A	6
B	5
D	4 ← Elim.

⇝ New Pref. Sched.

	6	9
1^{st}	A	B
2^{nd}	B	A

⇝

Cand.	# 1^{st} Place
A	6 ← Elim
B	9

$\boxed{\text{B wins}}$

Since the withdrawal of the losing candidate C alters the outcome, this violates IIAC, independence of irrelevant alternatives.

Name: ______________________ Problem 2: ____________ /25

Problem 2(25 points) In a law firm with one founder, F, two junior partners, P and Q, and one associate, A, a winning coalition is made up of either the founder and at least one junior partner, or both junior partners and the associate. Here is the complete list of all **winning coalitions**.

$$\{F, P, Q, A\},\ \{F, P, Q\},\ \{F, P, A\},\ \{F, Q, A\},\ \{P, Q, A\},\ \{F, P\},\ \{F, Q\}.$$

(a)(5 points) In each winning coalition listed above, determine every **critical player**. Indicate your answer clearly by underlining in the above list or by listing winning coalitions and critical players below. You need not show work for this part.

r	Coalition	# Critical
2	{F, P}, {F, Q}	F 2, P1, Q1, A0
3	{F, P, Q}, {F, P, A}, {F, Q, A}, {P, Q, A}	F 3, P2, Q2, A1
4	{F, P, Q, A}	F0, P0, Q0, A0

(b)(10 points) Compute the **Banzhaf number** of each player, compute the total, and then compute the **Banzhaf index** of each player. Leave the Banzhaf index in the form of a fraction. **Show all your work.**

2	1	1	0	5
3	2	2	1	3
0	0	0	0	3
				1

B_F: 5, B_P: 3, B_Q: 3, B_A: 1, Total: 12, β_F: 5/12, β_P: 3/12, β_Q: 3/12, β_A: 1/12.

(c)(10 points) Compute the **Shapley-Shubik number** of each player, compute the total, and then compute the **Shapley-Shubik index** of each player (left as a fraction). You may compute this either by listing all **sequential coalitions** together with **pivotal players** or by counting the number of sequential coalitions associated to every winning coalition with specified critical player. Either way, **show all your work.**

r	Coalition w/ Crit. Play.	Seq. Coal. w/ Pivotal Play.	
2	{F, P}	(P, F, Q, A), (P, F, A, Q)	F 2
2	{F, P}	(F, P, Q, A), (F, P, A, Q)	P 2
2	{F, Q}	(Q, F, P, A), (Q, F, A, P)	F 2
2	{F, Q}	(F, Q, P, A), (F, Q, A, P)	Q 2

r			
3	{F, P, Q}	(P, Q, F, A), (Q, P, F, A)	F 2
3	{F, P, A}	(P, A, F, Q), (A, P, F, Q)	F 2
3	{F, P, A}	(F, A, P, Q), (A, F, P, Q)	P 2
3	{F, Q, A}	(Q, A, F, P), (A, Q, F, P)	F 2
3	{F, Q, A}	(F, A, Q, P), (A, F, Q, P)	Q 2
3	{P, Q, A}	(Q, A, P, F), (A, Q, P, F)	P 2
3	{P, Q, A}	(P, A, Q, F), (A, P, Q, F)	Q 2
3	{P, Q, A}	(P, Q, A, F), (Q, P, A, F)	A 2

10
6
6
2

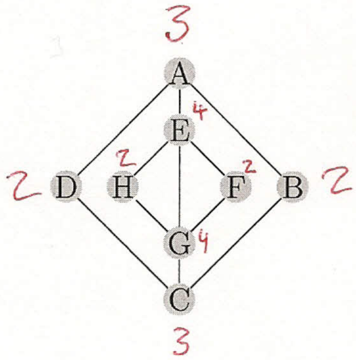
SS_F: 10, SS_P: 6, SS_Q: 6, SS_A: 2, Total: 24, σ_F: 10/24, σ_P: 6/24, σ_Q: 6/24, σ_A: 2/24

10/24 6/24 6/24 2/24

r	SS #
2	F4, P2, Q2, A0
3	F6, P4, Q4, A2
	F10, P6, Q6, A2

Name: ______________________________ **Problem 3**: ____________ /25

Problem 3(25 points)



(a)(5 points) For the graph above, list the **degrees** of all eight vertices.

A: 3, B: 2, C: 3, D: 2, E: 4, F: 2, G: 4, H: 2.

(b)(5 points) List all **odd vertices**, and also state the total number of odd vertices.

A & C. There are 2 odd vertices.

(c)(5 points) State whether not this graph has an **Euler cycle**, including a justification (if you use a result from the book, that is adequate justification, but you should give the correct statement of the result). By Euler's theorem for cycles, a connected graph has an Euler cycle if and only if there are zero odd vertices. Since this graph has two odd vertices, there is not an Euler cycle.

(d)(5 points) State whether not this graph has an **Euler path**, including a justification (if you use a result from the book, that is adequate justification, but you should give the correct statement of the result). Recall that in our definition, the start vertex of the Euler path is always different from the stop vertex. In case there is an Euler path, also list the vertices which will be the start and stop. By Euler's theorem for paths, a connected graph has an Euler path if & only if there are two odd vertices, in which case those vertices are the start & stop. So there is an Euler path with start/stop at A & C.

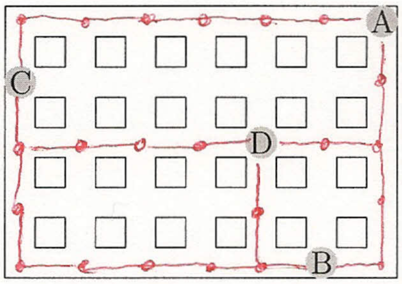
(e)(5 points) Find an **optimal Eulerization** of this graph. List the existing edge or existing edges which should be doubled in your optimal Eulerization.

One optimal Eulerization doubles the two edges AD & DC.
The other optimal Eulerization doubles the two edges AB & BC.

(You were only asked to find one.)

Name: ______________________ **Problem 4:** __________ /25

Problem 4(25 points)



The grid above shows the streets in a taxi driver's zone. The driver must find a **Hamiltonian cycle** beginning and ending at the intersection A, which crosses the intersections B, C and D, and which traverses the shortest distance both horizontally plus vertically (each small square is a city block).

(a)(5 points) Fill in the following distance chart, where units are city blocks travelled (horizontally plus vertically).

	A	B	C	D
A	*	5	7	4
B	5	*	8	3
C	7	8	*	5
D	4	3	5	*

(b)(10 points) Find one Hamiltonian cycle using the **nearest neighbor** method beginning at A, and compute the total length of this cycle. **Show all your work.**

$NN_A = D$. $A \xrightarrow{4} D$

$NN_D(\text{not } A) = B$. $A \xrightarrow{4} D \xrightarrow{3} B$

$NN_B(\text{not } A,D) = C$. $A \xrightarrow{4} D \xrightarrow{3} B \xrightarrow{8} C$

Close the cycle: $\boxed{A \xrightarrow{4} D \xrightarrow{3} B \xrightarrow{8} C \xrightarrow{7} A}$

Total length: $4 + 3 + 8 + 7 = \boxed{22}$

(c)(10 points) Find one Hamiltonian cycle using the **nearest neighbor** method beginning at C, and compute the total length of this cycle. **Show all your work.**

$NN_C = D$. $C \xrightarrow{5} D$

$NN_D(\text{not } C) = B$. $C \xrightarrow{5} D \xrightarrow{3} B$

$NN_B(\text{not } C,D) = A$. $C \xrightarrow{5} D \xrightarrow{3} B \xrightarrow{5} A$

Close the cycle: $\boxed{C \xrightarrow{5} D \xrightarrow{3} B \xrightarrow{5} A \xrightarrow{7} C}$

Total length: $5 + 3 + 5 + 7 = \boxed{20}$