MAT 514 Summer II 2019, SAMPLE MIDTERM 1,
Actual midterm is 3:30-4:30, August 1st in Earth and Space 181

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THIS EXAM IS WORTH 50 POINTS. QUESTIONS 1-40 ARE WORTH 1 POINT EACH. CHOOSE AND ANSWER 2 OF 41-44. EACH IS WORTH 5 POINTS. NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED.

1-10 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(1) T $(2-3i)-(4+2i)=-2-5i$

(2) T $(2+i)(3+i)=5+5i$

(3) F $(1-i)^3=-1-i.$

(4) F $1/i=i$

(5) F $e^{\pi i/4}=\sqrt{2}(1+i)$

(6) T $e^{100\pi i}=1$

(7) F $\frac{i}{2-i}=\frac{1+2i}{3}$

(8) F $\mathrm{Log}(-1)=\pi$

(9) F $Arg(1+i)=\frac{9\pi}{4}$

(10) T $e^i=\cos(1)+i\sin(1).$

11-15 Place the letter of the corresponding point in the box. The same letter might be used more than once.

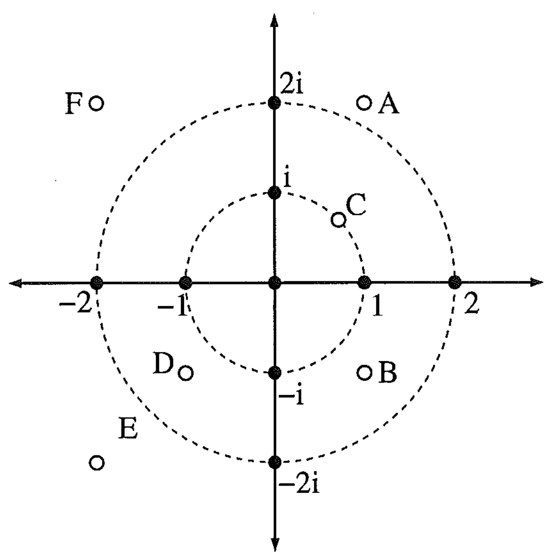
(11) A $|z|=\sqrt{5}$

(12) D $\mathrm{Re}(z)=-1.$

(13) B $z^2=-2i$

(14) E $z=\overline{F}$

(15) B $\mathrm{Arg}(z)=-\pi/4.$



16-20 Match each function with its definition. Assume $z = x + iy$.

(16) $\boxed{K}$ $\sinh(z)$

(17) $\boxed{I}$ $\exp(z)$

(18) $\boxed{A}$ $\sin(z)$

(19) $\boxed{C}$ $\tan(z)$

(20) $\boxed{M}$ i^z

A. $\frac{1}{2i}(e^{iz} - e^{-iz})$

B. $\frac{1}{2}(e^{iz} + e^{-iz})$

C. $(-i)\frac{e^{iz}-e^{-iz}}{e^{iz}+e^{-iz}}$

D. $\frac{e^{iz}+e^{-iz}}{e^{iz}-e^{-iz}}$

E. $\frac{1}{2}(e^{z} + e^{-z})$

F. $e^{y}(\cos x + i\sin x)$

G. $e^{x}(\cos x - i\sin x)$

H. $e^{x}\cos(y)$

I. $e^{x}\cos(y) + ie^{x}\sin(y)$

J. $e^{z\log i}$

K. $\frac{1}{2}(e^{z} - e^{-z})$

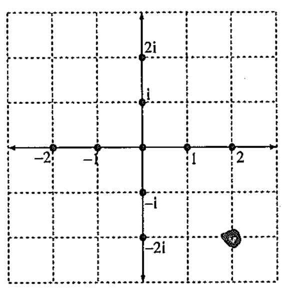
L. $\frac{1}{2}\log\frac{1+z}{1-z}$

M. $e^{i\log z}$

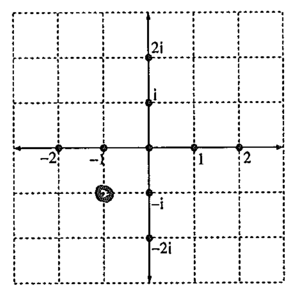
N. none of the above

21-25 Draw the following points or regions as accurately as you can.

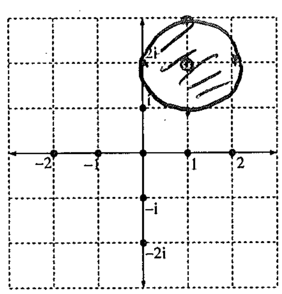
(21) Draw the point $z = 2 - 2i$.



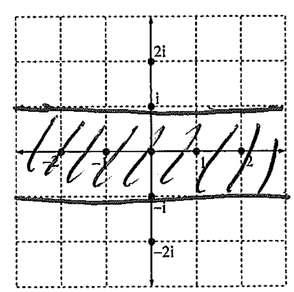
(22) Draw the point $\overline{iz}$, where $z = 1 + i$.



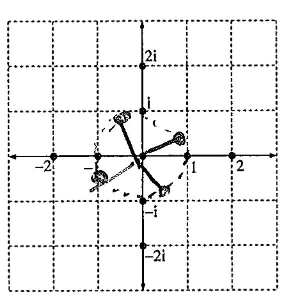
(23) Draw the region $|z + 1 - 2i| \leq 1$.



(24) Draw the region $|\mathrm{Im}(z)| \leq 1$.



(25) Draw all solutions of $z^4 = i$



21-30 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(26) $\boxed{\text{T}}$ The function e^z is entire.

(27) $\boxed{\text{T}}$ If $f = u + iv$ is holomorphic and real valued, then f must be constant.

(28) $\boxed{\text{F}}$ The path $\gamma(t) = t^3 + it^2$ is a smooth path.

(29) $\boxed{\text{F}}$ Any Mobius transformation is the composition of translations and inversions.

(30) $\boxed{\text{F}}$ The function $f(z) = (\bar{z})^2$ is holomorphic at 0.

(31) $\boxed{\text{T}}$ The function $\tan(z)$ is holomorphic on $\{z : |z| < 1\}$.

(32) $\boxed{\text{T}}$ For any complex numbers z and w, $|z - w| \geq |z| - |w|$.

(33) $\boxed{\text{F}}$ $f(x + iy) = 2xy + i(x^2 - y^2)$ is an entire function.

(34) $\boxed{\text{F}}$ Suppose $f = u + iv$. If the partials of u and v exist at a point z_0 and satisfy the Cauchy-Riemann equations at z_0, then f is differentiable at z_0 .

(35) $\boxed{\text{F}}$ An accumulation point for a set G can never be an interior point of G.

36-40: Give a precise statement of each definition or result.

(36) Define "Mobius transformation".

A function $f: \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ w/

$f(z) = \frac{az+b}{cz+d}$ $a, b, c, d \in \mathbb{C}$, $ad-bc \neq 0$.

(37) Define "$f : \mathbb{C} \to \mathbb{C}$ is complex differentiable at z_0".

$f: \mathbb{C} \to \mathbb{C}$ is complex differentiable at z_0 if

$$f'(z_0) := \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

exists.

(38) State De Moivre's Theorem

n a natural number.

(1) $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

OR

(2) $(e^{i\theta})^n = e^{in\theta}$

(39) Define "smooth path".

A path $\gamma: [a,b] \to \mathbb{C}$ is smooth if

(1) γ continuous

(2) $\gamma'(z) \neq 0$, $z \in [a,b]$

(3) $\gamma'(t)$ exists and is continuous

(40) Define "$z \in \mathbb{C}$ is a boundary point of G"

$z \in \mathbb{C}$ is a boundary point of G if for all $r > 0$, $B(z, r)$ contains points in G and not in G.

41-44: Answer two of the following questions. Mark clearly which questions you are answering

(41) Give an example of a Mobius transformation taking $1 \mapsto 5$, $\infty \mapsto i$, and $0 \mapsto 0$.

(42) Suppose f is entire and the image of f is contained inside of the imaginary axis. Prove that f must be constant.

(43) Find a piecewise smooth parameterization of a triangle with vertices $1, -1$, and i begininng at 1 oriented clockwise.

(44) Sketch the set of points z so that $|z| = Re(z) + 1$. Describe your sketch in terms of a familiar geometric object, and prove that your sketch is correct.

(41) $f(z) = [z, 1, \infty, 0]$ $\qquad$ $g(z) = [z, 5, i, 0]$

$1 \to 0$ $\qquad$ $5 \to 0$

$\infty \to 1$ $\qquad$ $i \to 1$

$0 \mapsto \infty.$ $\qquad$ $0 \to \infty.$

$h = g^{-1} \circ f$ is a Mobius transformation that works, since composition of Möbius transformations are Möbius.

(42) $f = u + iv : \mathbb{C} \to \mathbb{C}$. entire.

$f = iv$ since image is imaginary axis.

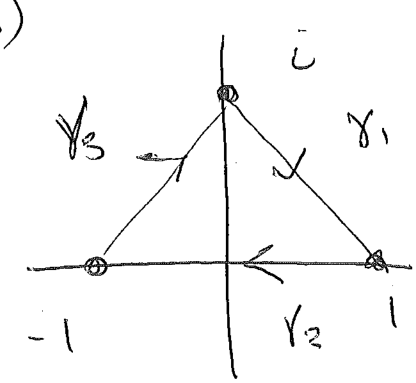
C-R eqn's hold.

$u_x = v_y$ $\qquad$ $u_x = 0 \Rightarrow v_y = 0$ $\qquad$ v constant

$u_y = -v_x$ $\qquad$ $u_y = 0 \Rightarrow v_x = 0$ $\qquad$ $f = iv$ constant.

Additional Page 3

(43)



Line segment from z to w:
$\gamma(t) = z - t(w - z)$

$\gamma_1 : [0,1] \to \mathbb{C}$

$\gamma_1(t) = i + t \cdot (1 - i) = i + t(1-i)$

$\gamma_2 : [1,2] \to \mathbb{C}$

$\gamma_2(t) = 1 + (t-1)(-1-1)$

$= 1 - 2(t-1)$

$\gamma_3 : [2,3] \to \mathbb{C}$

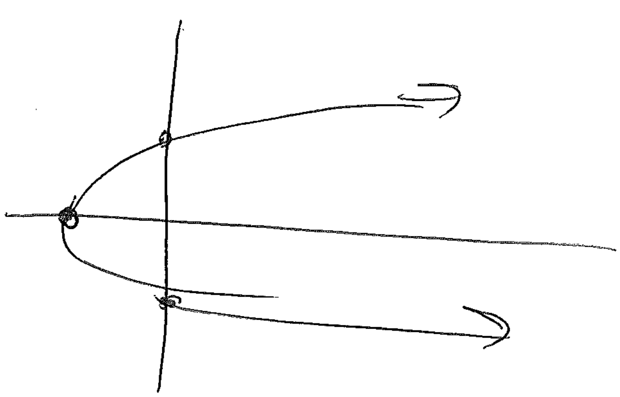
$\gamma_3(t) = -1 + (t-2)(i+1)$

(44) If $z = x + iy$.

$|z|^2 = (\mathrm{Re}(z) + 1)^2 = \mathrm{Re}(z)^2 + 2\mathrm{Re}(z) + 1$

$\Rightarrow x^2 + y^2 = x^2 + 2x + 1$

$y^2 = 2x + 1$



$y = 0 \iff x = -\frac{1}{2}$

$x = 0$ iff $y^2 = 1$ $y = \pm 1$

Shape is a parabola.

Commentary on other problems.

~~Additional Page 1~~

#1 $(2-3i)-(4+2i) = (2-4)+i(-3-2) = -2-5i$

#2 $(2+i)(3+i) = 6-1 \quad +i(3+2) = 5+5i$

#3 False. $|1-i| = \sqrt{2}$, $|(1-i)^3| = (\sqrt{2})^3 = 2\cdot\sqrt{2}$

$|-1-i| = \sqrt{2}$

#4 $\frac{1}{i} = \frac{1}{i}\cdot\frac{i}{i} = \frac{i}{-1} = -i$

(#5) False. $|e^{i\pi/4}| = 1$, $|\sqrt{2}(1+i)| > 1$.

#6 100π is a multiple of 2π

#7 $\frac{i}{2-i}\cdot\frac{2+i}{2+i} = \frac{2i-1}{5}$

#8 $\mathrm{Arg}(z) \in [-\pi, \pi)$.

#9 $\mathrm{Arg}(z) \in [-\pi, \pi)$, $9\pi/4$ isn't

#10 $e^{i\theta} = \cos\theta + i\sin\theta$, $\theta = 1$.

(11)-(14)

All clear. Use the geometric version of multiplication to see that $B^2 = -2i$.

(16) - (20)

Refer to definitions in notes or in chapter 3.

(22) $iz = i - 1$

$\overline{iz} = -i - 1$

(23) $|z + 1 - 2i| = |z - (1+2i)|$.

(24) $z = x + iy \Rightarrow \operatorname{Im}(z) = y$.

$|y| \leq 1$.

(25) $|z| = 1$

$4\theta = \frac{\pi}{2} + 2\pi n$. $\theta = \frac{\pi}{8} + \frac{\pi}{2} n$.

$\theta = \frac{\pi}{8}$ $\theta = \frac{\pi}{8} + \frac{\pi}{2}$ $\theta = \frac{\pi}{8} + \pi$ $\theta = \frac{\pi}{8} + \frac{3\pi}{2}$.

4 total solutions, all equally spaced.

21 - 30

(27) See HW 2.20, or use.

$f = u$

$u_x = v_y = 0$ $\quad$ u constant

$u_y = v_x = 0$ $\quad$ $\Rightarrow f$ constant

(28) $\gamma'(t) = 3t^2 + i \cdot 2t$ $\quad$ Smooth only

$\gamma'(0) = 0$ $\quad$ if $\gamma' \neq 0$.

(29) Also dilations!

(30) No, b/c

(1) holomorphic at a point makes no sense

(2) Not differentiable on $\mathbb{C} \setminus \{0\}$.

(31) $\tan(z) = \dfrac{\sin(z)}{\cos(z)}$

not in $|z| < 1$.

$\cos z = 0$ only if $z = \frac{\pi}{2} + n\pi$ $\quad n \in \mathbb{Z}$.

sin, cos holomorphic, so $\tan(z)$ is

(32) This is the reverse triangle inequality.

(33) $u_x = 2y \qquad v_x = 2x$ C-R do not hold

$u_y = 2x \qquad v_y = -2y$ for all $z \in \mathbb{C}$

<u>Not</u> entire

(34) No mention of continuity of ~~[illegible]~~

u_x, u_y, v_x, v_y at z_0.

(35). Actually interior points <u>always</u> are accumulation points