MAT 514 Summer II 2019, SAMPLE MIDTERM 1,
Actual midterm is 3:30-4:30, August 1st in Earth and Space 181

Name	ID	

THIS EXAM IS WORTH 50 POINTS. QUESTIONS 1-40 ARE WORTH 1 POINT EACH. CHOOSE AND ANSWER 2 OF 41-44. EACH IS WORTH 5 POINTS. NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED.

1-10 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(1) $(2-3i)-(4+2i)=-2-5i$

(2) $(2+i)(3+i)=5+5i$

(3) $(1-i)^3=-1-i.$

(4) $1/i=i$

(5) $e^{\pi i/4}=\sqrt{2}(1+i)$

(6) $e^{100\pi i}=1$

(7) $\frac{i}{2-i}=\frac{1+2i}{3}$

(8) $\mathrm{Log}(-1)=\pi$

(9) $Arg(1+i)=\frac{9\pi}{4}$

(10) $e^i=\cos(1)+i\sin(1).$

11-15 Place the letter of the corresponding point in the box. The same letter might be used more than once.

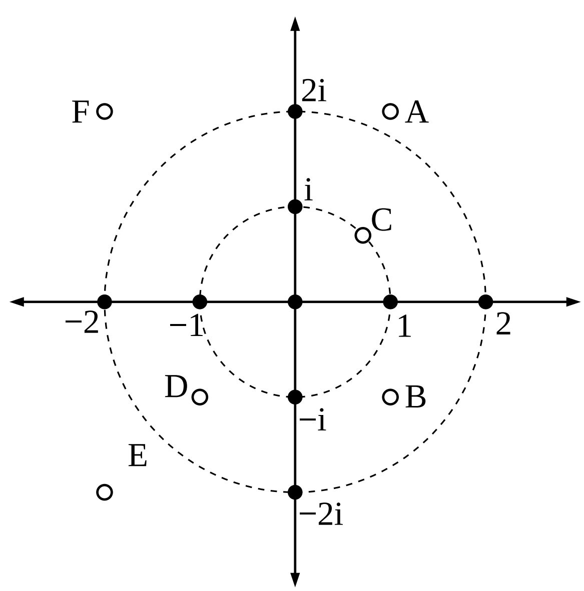
(11) $|z|=\sqrt{5}$

(12) $\mathrm{Re}(z)=-1.$

(13) $z^2=-2i$

(14) $z=\overline{F}$

(15) $\mathrm{Arg}(z)=-\pi/4.$



16-20 Match each function with its definition. Assume $z = x + iy$.

(16) ☐ $\sinh(z)$

(17) ☐ $\exp(z)$

(18) ☐ $\sin(z)$

(19) ☐ $\tan(z)$

(20) ☐ i^z

A. $\frac{1}{2i}(e^{iz} - e^{-iz})$

B. $\frac{1}{2}(e^{iz} + e^{-iz})$

C. $(-i)\frac{e^{iz}-e^{-iz}}{e^{iz}+e^{-iz}}$

D. $\frac{e^{iz}+e^{-iz}}{e^{iz}-e^{-iz}}$

E. $\frac{1}{2}(e^{z} + e^{-z})$

F. $e^{y}(\cos x + i \sin x)$

G. $e^{x}(\cos x - i \sin x)$

H. $e^{x}\cos(y)$

I. $e^{x}\cos(y) + ie^{x}\sin(y)$

J. $e^{z \log i}$

K. $\frac{1}{2}(e^{z} - e^{-z})$

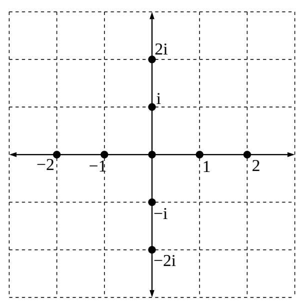
L. $\frac{1}{2}\log\frac{1+z}{1-z}$

M. $e^{i \log z}$

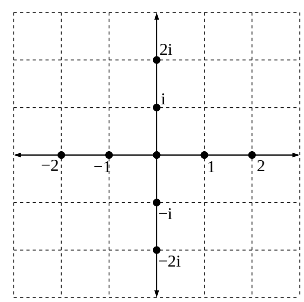
N. none of the above

21-25 Draw the following points or regions as accurately as you can.

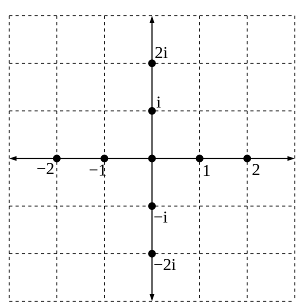
(21) Draw the point $z = 2 - 2i$.



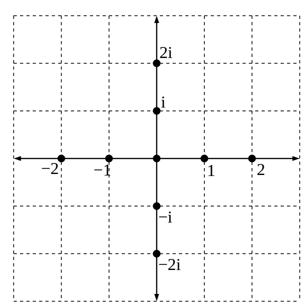
(22) Draw the point $\overline{iz}$, where $z = 1 + i$.



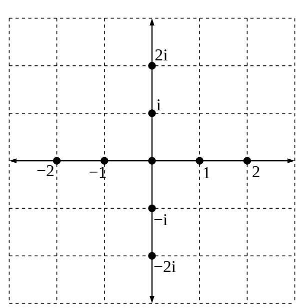
(23) Draw the region $|z + 1 - 2i| \leq 1$.



(24) Draw the region $|\text{Im}(z)| \leq 1$.



(25) Draw all solutions of $z^4 = i$



21-30 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(26) ☐ The function e^z is entire.

(27) ☐ If $f = u + iv$ is holomorphic and real valued, then f must be constant.

(28) ☐ The path $\gamma(t) = t^3 + it^2$ is a smooth path.

(29) ☐ Any Mobius transformation is the composition of translations and inversions.

(30) ☐ The function $f(z) = (\bar{z})^2$ is holomorphic at 0.

(31) ☐ The function $\tan(z)$ is holomorphic on $\{z : |z| < 1\}$.

(32) ☐ For any complex numbers z and w, $|z - w| \geq |z| - |w|$.

(33) ☐ $f(x + iy) = 2xy + i(x^2 - y^2)$ is an entire function.

(34) ☐ Suppose $f = u + iv$. If the partials of u and v exist at a point z_0 and satisfy the Cauchy-Riemann equations at z_0, then f is differentiable at z_0 .

(35) ☐ An accumulation point for a set G can never be an interior point of G.

36-40: Give a precise statement of each definition or result.

(36) Define “Mobius transformation”.

(37) Define “$f : \mathbb{C} \to \mathbb{C}$ is complex differentiable at z_0”.

(38) State De Moivre’s Theorem

(39) Define “smooth path”.

(40) Define “$z \in \mathbb{C}$ is a boundary point of G”

41-44: Answer two of the following questions. Mark clearly which questions you are answering

(41) Give an example of a Mobius transformation taking $1 \mapsto 5$, $\infty \mapsto i$, and $0 \mapsto 0$.

(42) Suppose f is entire and the image of f is contained inside of the imaginary axis. Prove that f must be constant.

(43) Find a piecewise smooth parameterization of a triangle with vertices $1, -1$, and i begininng at 1 oriented clockwise.

(44) Sketch the set of points z so that $|z| = Re(z) + 1$. Describe your sketch in terms of a familiar geometric object, and prove that your sketch is correct.

Additional Page 1

Additional Page 2

Additional Page 3