MAT514 - Analysis for Teachers II - Summer II 2019

Homework 5

Due Thursday August 15. Updated August 9

Chapter 7 Problems: 7.25; 7.26(bc); 7.33(beg); 7.34.

Chapter 8 Problems: 8.10(bc); 8.28(ab); 8.29 (HINT: $\sin(z)$ is zero ir and only if $z = \pi n$ for some integer n. $\cos(z) = 0$ if and only if $z = \pi/2 + \pi n$ for some integer n.)

Chapter 9 Problems: 9.2(ace); 9.6; 9.7(ac); 9.8(ace); 9.15.

Additional Problems (25 Total Extra Credit Points):

1. Suppose that $f : \mathbb{C} \to \mathbb{C}$ is non-constant, meromorphic, and satisfies $f(z) = f(z+1)$ and $f(z) = f(z+i)$ (we say f is doubly periodic). Use Liouville's theorem to deduce that f must have a pole in the square S with vertices $(0, 1, i, 1+i)$ (HINT: Suppose f had no poles, so that it is entire. You may use the fact that if f is a continuous function on a closed and bounded set, it is bounded. Show that because of the periodicity relationship, f is completely determined by its values on the unit square $z = x + iy, x, y \in [0, 1]$).

2. Let f be as above. Can f have exactly one simple pole in S, with no other poles? (HINT: The residue of the simple pole can be calculated by integrating along the square S. On the other hand, what happens when you apply the periodicity relations to this integral?).