MAT514 - Analysis for Teachers II - Summer II 2019

Homework 4
Due Thursday August 8th

Chapter 4 Problems: 4.4 (do the general case only using the definition of the integral. Remember the parameterization is $\gamma(t) = w + re^{it}$ with $t \in [0, 2\pi]/)$: 4.5 (ac) (use the definition of the integral); 4.7 (abc) (HINT: Does $\exp(3z)$ have an antiderivative on $\mathbb{C}$? Use any theorem you like from the course to answer this question); 4.8 (acf) (again, use whatever theorem from the course to make this as easy as possible, but explain which results you are using); 4.18(a) (HINT: simplify the integrand. Does the integrand have an antiderivative?); 4.37 (Use the Cauchy Integral Formula, and any other useful trick like partial fractions).

Chapter 5 Problems: 5.1 (ac); 5.3(cdh) (HINT: For c and h use partial fractions. For h, factor $z^2 + 1$). 5.14 (HINT: Consider the function 1/f).

Additional Problems:

1. Let $n \geq 2$ be a positive integer. Does

   $$f(z) = \frac{1}{z^n}$$

   have an antiderivative on $\mathbb{C} \setminus 0$? Compute

   $$\int_\gamma \frac{1}{z^n}\, dz$$

   for $\gamma = C(0, 1)$, oriented counterclockwise. Compare to the $n = 1$ case studied in class.

2. Let a be a complex number. Show that if $r < |a|$, then

   $$\int_{C(0,r)} \frac{1}{z-a}\, dz = 0.$$

   Show that if $r > |a|$, then

   $$\int_{C(0,r)} \frac{1}{z-a}\, dz = 2\pi i.$$

   State clearly any theorem from the course you use to do this problem.

3. Show that $1/z$ is holomorphic on an annulus $A(1, 2) = \{z : 1 < |z| < 2\}$. Compute $\int_{C[0,3/2]} \frac{1}{z}\, dz$. Is this a contradiction the Cauchy's Theorem? Explain