MAT 514 Summer II 2019, SAMPLE FINAL 1,
Actual midterm is 3:30-4:30, August 1st in Earth and Space 181

Name	ID	

THIS EXAM IS WORTH 50 POINTS. EACH QUESTION IS WORTH 1 POINT, EXCEPT FOR THE QUESTIONS 31-35, IS WORTH 1 POINT. QUESTIONS 31-35 ARE WORTH TWO POINTS EACH. NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED.

1-5 Calculate the order of the zero or the pole at the given point. Write 0 if the function does not have a zero at the given point

(1) ☐ $f(z) = \exp(z) - 1 - z^2/2$ when $z = 0$

(2) ☐ $f(z) = 1 + z + z^2 + \dots$ when $z = 0$.

(3) ☐ $f(z) = \frac{1}{\sin(z) - z}$ when $z = 0$.

(4) ☐ $f(z) = \sum_{k=5}^{\infty} c_k (z - z_0)^k$ when $z = z_0$.

(5) ☐ $f(z) = \sin(z) - 1$ at $z = \pi/2$.

6-10: Calculate the residue of each function at the given point.

(6) ☐ $f(z) = \exp(z)$ when $z = z_0 \in \mathbb{C}$.

(7) ☐ $f(z) = \dots \frac{1}{z^2} - \frac{1}{z} + 1 - z + z^2 + \dots$ when $z = 0$.

(8) ☐ $f(z) = \frac{1}{z(z^2+1)}$ when $z = -i$.

(9) ☐ $f(z) = \frac{1}{z^2}$ when $z = 0$.

(10) ☐ $f(z) = \frac{1}{\sin(z)}$ when $z = 0$.

11-15 Match each function with its power series expansion.

(11) ☐ $\frac{1}{(1-z)^2}$

(12) ☐ $z\sin(z)$

(13) ☐ $\exp(z^2)$

(14) ☐ $\frac{1}{4-z}$

(15) ☐ $\frac{\sin(z)}{z}$

A. $\sum_{k=0}^{\infty} z^k$

B. $\sum_{k=1}^{\infty} kz^{k-1}$

C. $\sum_{k=1}^{\infty} kz^k$

D. $\sum_{k=0}^{\infty}(-1)^k \frac{z^{2k+2}}{(2k+1)!}$

E. $\sum_{k=0}^{\infty}(-1)^k \frac{z^{2k+1}}{(2k)!}$

F. $\sum_{k=0}^{\infty} 4^{-k} z^k$

G. $\sum_{k=0}^{\infty} 4^{-k-1} z^k$

H. $\sum_{k=0}^{\infty} 2^{-k-1} z^k$

I. $\sum_{k=0}^{\infty} 2^{-k} z^k$

J. $\sum_{k=0}^{\infty}(-1)^k \frac{z^{2k}}{(2k+1)!}$

K. $\sum_{k=0}^{\infty} \frac{z^{(k^2)}}{k!}$

L. $\sum_{k=0}^{\infty} \frac{z^{2k}}{k!}$

M. $\sum_{k=0}^{\infty} \frac{z^k}{k!}$

N. none of the above

16-20: Identify the radius of convergence of the following power series

(16) ☐ $\sum_{k=0}^{\infty} z^k$

(17) ☐ $\sum_{k=1}^{\infty} kz^{k-1}$

(18) ☐ $\sum_{k=0}^{\infty} \frac{(z+2)^k}{k^k}$

(19) ☐ $\sum_{k=0}^{\infty} 6^k (z-1)^k$

(20) ☐ $\sum_{k=0}^{\infty} k^2 (z+1)^k$

21-30 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(21) ☐ Cauchy's theorem implies that if γ is a smooth simple closed curve, and f is holomorphic on a domain U containing γ, then $\int_\gamma f = 0$.

(22) ☐ The function $f(z) = 1/z$ has an antiderivative on the domain $\mathbb{C} \setminus \{0\}$

(23) ☐ If U is a simply connected region. and f is a holomorphic function, f has an antiderivative on U.

(24) ☐ A power series centered at z_0 with radius of convergence R is holomorphic on $D(z_0, R)$.

(25) ☐ If $\sum_{k=0}^{\infty} c_k z^k$ has radius of convergence 1, the series $\sum_{k=0}^{\infty} c_k$ always converges.

(26) ☐ The function $\exp(1/z)$ has a pole of infinite order at $z = 0$.

(27) ☐ A function is holomorphic if and only if it is analytic.

(28) ☐ When integrating a function f along a smooth path γ, the orientation of γ does not effect the result.

(29) ☐ If $f(z)$ is not an entire function, and γ is a smooth, simple, closed path, then $\int_\gamma f$ can never be 0.

(30) ☐ Suppose $f(z)$ has a pole of order two at z_0. Then the residue of $f(z)$ at $z = z_0$ is $\lim_{z \to z_0} \frac{d}{dz} \left((z - z_0)^2 f(z)\right)$

31-35: Give a precise statement of each definition or result. Each question is worth two points

(31) Let $\gamma : [a, b] \to \mathbb{C}$ be a smooth path, and let f be continuous. Define "integral of f with respect to γ.

(32) State Cauchy's Theorem. Include all hypotheses.

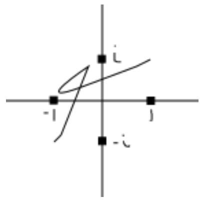
(33) Let $f : U \to \mathbb{C}$ be a continuous function. Define "F is the antiderivative of f"

(34) Define "power series centered at $z_0 = -1$".

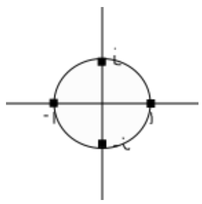
(35) State the Cauchy Integral formula. Include all hypotheses

36-45: Calculate the following integrals. Unless otherwise stated, assume the path is oriented counter clockwise (i.e, positively oriented.)

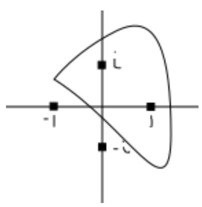
(36) $\int_\gamma z\,dz.$



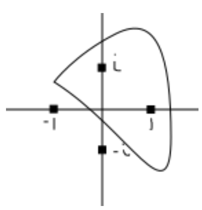
(37) $\int_\gamma (\bar{z})^2\,dz.$



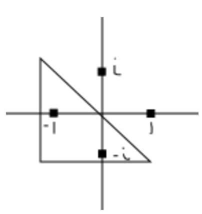
(38) $\int_\gamma \cos(z)\,dz.$



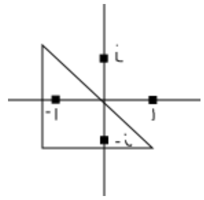
(39) $\int_\gamma \frac{\cos(z)}{z-1}\,dz.$



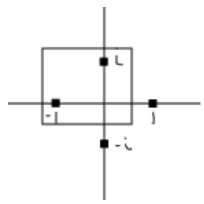
(40) $\int_\gamma \frac{1}{(z-i)(z+1)}\,dz.$



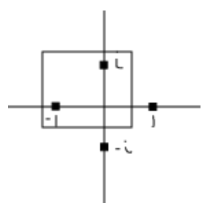
(41) $\square$ $\int_\gamma \frac{1}{(z+i)(z+1)}\,dz.$



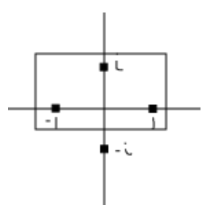
(42) $\square$ $\int_\gamma \frac{1}{z(z+i)(z+1)}\,dz.$



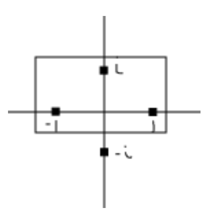
(43) $\square$ $\int_\gamma \frac{1}{z^3}\,dz.$



(44) $\square$ $\int_\gamma \sum_{k=-\infty}^{0} \frac{1}{(-k+1)!} z^k\,dz.$



(45) $\square$ $\int_\gamma \frac{\cos(z)-1}{z^3}\,dz.$



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