MAT 319 Quiz #4 with solutions Wednesday 4/2/03

Determine whether each of the following four assertions is TRUE or FALSE. Give a brief explanation for your answer.

1. If f is a continuous function on $(0,1)$, then it is bounded.
SOLUTION FALSE. Look at $f(x) = \frac{1}{x}$.

2. If f is uniformly continuous on $(0,1)$, then it is bounded.
SOLUTION TRUE. Choose $\epsilon = 1$, for example. There is a $\delta > 0$, such that $|f(x) - f(y)| < 1$ whenever $|x - y| < \delta$. We may and do assume that $\delta < 1$. Consider the points $x_1 = \delta, x_2 = 2\delta, ..., x_n = n\delta$ in the interval $(0,1)$, where n is the largest positive integer such that $n\delta < 1$. Every point $x \in (0,1)$ is within δ of one of the x_i. Hence $1 + \max\{f(x_1), f(x_2), ..., f(x_n)\}$ is a bound for $|f(x)|$ for all x.

3. If f is integrable on $[0,1]$, then it is continuous on $(0,1)$.
SOLUTION FALSE. The function $f(x) = \begin{cases} 0 & \text{for } 0 \le x < \frac{1}{2} \\ 1 & \text{for } \frac{1}{2} \le x \le 1 \end{cases}$ is a counterexample.
4. If f is a differentiable function on $[0,1]$, then it is integrable on $[0,1]$.
SOLUTION TRUE because a differentiable function is continuous and a continuous function is integrable.

5. State (carefully) any form of the Fundamental Theorem of Calculus.
SOLUTION FFTC: If f is an integrable function on $[a,b]$ and if there exists a differentiable function g on $[a,b]$ such that $g' = f$, then

$$\int_a^b f = g(b) - g(a).$$

or
SFTC: If f is an integrable function on $[a,b]$ and we define for $x \in [a,b]$, $F(x) = \int_a^x f$. Then for all $x \in [a,b]$, $F'(x) = f(x)$.