1. If $a$ and $b$ are irrationals, then so is $a + b$.
**SOLUTION.** FALSE. $\sqrt{2} + (-\sqrt{2}) = 0$.

2. Let $a$ and $b$ be irrationals with $a < b$. There exists a rational $c$ and a positive integer $n$ such that $a < c + \frac{\sqrt{2}}{n} < b$.
**SOLUTION.** TRUE. There certainly exists a rational $c$ such that $a < c < b$. Since $\frac{\sqrt{2}}{n}$ is very small and positive for large $n$, we can choose such an $n$ so that $c + \frac{\sqrt{2}}{n} < b$.

3. Define carefully a Dedekind cut.
**SOLUTION.** A Dedekind cut $\alpha$ is a proper nonempty subset of the rationals such that
(i) whenever $p \in \alpha$ and $q$ is a rational $< p$, then $q \in \alpha$, and
(ii) for all $p \in \alpha$, there exists an $r \in \alpha$ with $r > p$.

4. State the least upper bound property.
**SOLUTION.** Every nonempty set of reals that is bounded from above has a least upper bound.

5. Let $S$ be a nonempty set of real numbers. Assume that $\alpha$ and $\beta$ are least upper bounds for $S$. Show that $\alpha = \beta$.
**SOLUTION.** Because $\alpha$ is an upper bound and $\beta$ is least upper bound, $\alpha \geq \beta$. Because $\alpha$ and $\beta$ are interchangeable, $\beta \geq \alpha$. The two inequalities say that $\beta = \alpha$. 