MAT 319 Quiz #1 Friday 1/31/03

Determine whether each of the following 2 assertions is TRUE or FALSE. Give a brief explanation for your answer.

1. If a and b are irrationals, then so is $a + b$.
SOLUTION. FALSE. $\sqrt{2} + (-\sqrt{2}) = 0$.

2. Let a and b be irrationals with $a < b$. There exists a rational c and a positive integer n such that $a < c + \frac{\sqrt{2}}{n} < b$.
SOLUTION. TRUE. There certainly exists a rational c such that $a < c < b$. Since $\frac{\sqrt{2}}{n}$ is very small and positive for large n, we can choose such an n so that $c + \frac{\sqrt{2}}{n} < b$.

3. Define carefully a Dedekind cut.
SOLUTION. A Dedekind cut α is a proper nonempty subset of the rationals such that
(i) whenever $p \in \alpha$ and q is a rational $< p$, then $q \in \alpha$, and
(ii) for all $p \in \alpha$, there exists an $r \in \alpha$ with $r > p$.

4. State the least upper bound property.
SOLUTION. Every nonempty set of reals that is bounded from above has a least upper bound.

5. Let S be a nonempty set of real numbers. Assume that α and β are least upper bounds for S. Show that $\alpha = \beta$.
SOLUTION. Because α is an upper bound and β is least upper bound, $\alpha \geq \beta$. Because α and β are interchangeable, $\beta \geq \alpha$. The two inequalities say that $\beta = \alpha$.