MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #8 with solutions

12/10/2002

Problems 1 & 2: True or false: (Circle the correct answers.) Let G be a group.

T F (1) If a and $b \in G$ and $r \in \mathbf{Z}$, then the equation $a^r x = b$ has a unique solution $x \in G$.

T F (2) If $(ab)^r = a^r b^r$ for all a and $b \in G$ and all $r \in \mathbf{Z}$, then G is abelian.

SOLUTION: (1) is TRUE, $x = a^{-r}b$.
(2) is also TRUE. The equation for $r = 2$ reads $abab = aabb$. Multiplying both sides on the left by a^{-1} and on the right by b^{-1} yields $ba = ab$.

Problem 3: The set K consisting of the four complex numbers $\{\pm 1, \pm \imath\}$ is a group under multiplication. It is isomorphic to $(\mathbb{Z}_4, +)$. Describe such an isomorphism $\theta : K \to \mathbb{Z}_4$ by specifying $\theta(1)$, $\theta(-1)$, $\theta(\imath)$ and $\theta(-\imath)$.

SOLUTION: We must map a generator (for example) $\imath$ of K to a generator (for example) $[1]_4$ of $\mathbb{Z}_4$. Hence $\theta(1) = [0]_4$, $\theta(-1) = [2]_4$, $\theta(\imath) = [1]_4$ and $\theta(-\imath) = [3]_4$.

Problem 4: Describe two non-isomorphic groups of order 6.

SOLUTION: The groups $\mathbb{Z}_6$ and $S(3)$ have order 6 and are not isomorphic since $\mathbb{Z}_6$ is abelian while $S(3)$ is not.

Problem 5: State (carefully) Lagrange's theorem.

SOLUTION: Let H be a subgroup of a finite group G. Then $o(H)$ divides $o(G)$.