Problems 1 & 2: True or false: (Circle the correct answers.) Let $X$, $Y$ and $Z$ be sets.

- T  F  (1) If $X \cap Y = \emptyset$, then either $X = \emptyset$ or $Y = \emptyset$.
- T  F  (2) $(X \cup Y)^c \cup Z = X^c \cup Y^c \cup Z$.

SOLUTION: (1) is FALSE. Let $X$ be the set of even integers and $y$, the set of odd integers.
(2) is also FALSE. The set $Z$ does not play any role in the problem, so you may take $Z = \emptyset$. Let $Y$ be a proper subset of $X$. So the statement in this case reads $X^c = Y^c$ which is clearly not true.

The next three problems involve the permutation $\pi = (1, 2, 3, 6)(1, 4, 5)(8, 9) \in S(10)$.

Problem 3: Write $\pi$ as a product of disjoint cycles.
SOLUTION: $\pi = (1, 4, 5, 2, 3, 6)(8, 9)$.

Problem 4: Write $\pi$ and $\pi^{-1}$ as products of transpositions.
SOLUTION: $\pi = (1, 6)(1, 3)(1, 2)(1, 5)(1, 4)(8, 9)$ and $\pi^{-1} = (8, 9)(1, 4)(1, 5)(1, 2)(1, 3)(1, 6)$.

Problem 5: What are the fixed points of $\pi$, the order of $\pi$, and the sign of $\pi$?
SOLUTION: The fixed points of $\pi$ are 7 and 10.

- $o(\pi) = \text{lcm}(6, 2) = 6$.
- $\text{sgn}(\pi) = (-1)^6 = 1$. 