Exercise 1. For each of these limits, say whether they exist, and if need be, precise the limit.

a. \( \lim_{x \to 0} \frac{\sin^2 x}{x} \)

b. \( \lim_{x \to 0^+} \frac{\tan x - 1}{x} \)

c. \( \lim_{x \to 0} \frac{x}{\sin x} \)

d. \( \lim_{x \to 0} x^6 \sin \left( \frac{1}{x} \right) \)

Exercise 2. Show that each of the following equations admit a solution \( x \) and give an interval of the form \([a, a + 1]\) to which the solution belongs:

a. \( 3x^4 - x^3 + 8x + 2 = 0 \)

b. \( \cos(3x) = \frac{1}{3} \)

Exercise 3. For each of these limits, say whether they exist, and if need be, precise the limit.

a. \( \lim_{x \to 5^+} \frac{x^4 + 2x + 1}{5 - x} \)

b. \( \lim_{x \to +\infty} \frac{2x^3 + x + 1}{x^3 - 3} \)

c. \( \lim_{x \to +\infty} \frac{2x^4 + x^2 - 3x + 1}{5x^4 - 3x + 1} \)

d. \( \lim_{x \to +\infty} \frac{6x^7 + 2x^3 - x + 2}{7x^7 - x + 13} \)

e. \( \lim_{x \to +\infty} \sqrt{x} - 7 \sin^2 x \)

f. \( \lim_{x \to 3} \frac{\sqrt{x} - 3}{x - 3} \) (for this one, give two different methods).

g. \( \lim_{x \to 7} \frac{x - 7}{\sqrt{x} - \sqrt{7}} \)

Exercise 4. For each of the following functions, precise the domain and compute the derivative:

a. \( \frac{x^3 + 3x^2 - x}{x^2 - 4} \)
b. $x^7e^x + \tan x$

c. $\sin x \cos x$

d. $\cos^2 x$

e. $e^{3x}$

f. $e^{-2x}$

**Exercise 5.** Let $f$ be the function defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 
  x + 1 & \text{if } x < 0 \\
  e^x & \text{if } x \geq 0 
\end{cases}$$

Show that $f$ is continuous on its whole domain of definition. What about differentiability?

**Exercise 6.** Give the equation of the tangent to the graph of $y = e^{3x} + x$ at $x = 0$. Same question with $y = x^3 + 2x + 3$ at $x = 1$. Sketch a graph.