MAT 211 Summer 2015 Homework 5

Due in class June 29$^{\text{th}}$.

Problem 1. Let V be the subspace of $\mathbb{R}^4$ spanned by the vector $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

Find a basis for the orthogonal complement $V^\perp$ of V.

Problem 2. Find a basis for $W^\perp$, where

$$W = span\left(\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \right).$$

Problem 3. Consider the vectors

$$\vec{u}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$$

in $\mathbb{R}^4$. Can you find a vector $\vec{u}_4$ in $\mathbb{R}^4$ such that the vectors $\vec{u}_1,\ \vec{u}_2,\ \vec{u}_3,\ \vec{u}_4$ are orthonormal? If so, how many such vectors are there?

Problem 4. Find an orthonormal basis for each of the following subspaces

a) $V = span\left(\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 3 \end{bmatrix} \right)$;

b) $V = span\left(\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} \right)$;

c) $V = span\left(\begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 1 \\ 6 \end{bmatrix} \right)$.

Problem 5. Find the QR factorizations of the following matrices. You may use your results of Problem 4.

a) $A = \begin{bmatrix} 6 & 2 \\ 3 & -6 \\ 2 & 3 \end{bmatrix}$;

b) $B = \begin{bmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}$;

c) $C = \begin{bmatrix} 1 & 0 & 1 \\ 7 & 7 & 8 \\ 1 & 2 & 1 \\ 7 & 7 & 6 \end{bmatrix}$.

Problem 6. Let V be the subspace of $\mathbb{R}^3$ given by the equation

$$x + 2y + 3z = 0.$$

a) Write out the formula of the orthogonal projection $proj_V(\vec{x})$ explicitly;

b) Use the formula you find to compute $proj_V\left(\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$;

c) Write out the matrix of the orthogonal projection under the standard basis of $\mathbb{R}^3$.

Problem 7. Use Gauss elimination to find the determinant of the following matrices.

a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 2 & 2 & 5 \end{bmatrix}$;

b) $\begin{bmatrix} 1 & -1 & 2 & -2 \\ -1 & 2 & 1 & 6 \\ 2 & 1 & 14 & 10 \\ -2 & 6 & 10 & 33 \end{bmatrix}$;

c) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{bmatrix}$.