Review sheet for MAT515, Fall 2018

Know the eight axioms, L1-L8.

Define the basic isometries. Are they all bijections? Is the inverse of a basic isometry also a basic isometry?

State the fundamental theorem of Similarity and prove it for the case that the scale factor is $1/2$.

Define a convex set and a convex angle.

Let F and G be transformations of the plane. Is it necessarily true that $F \circ G = G \circ F$? Is it true if F and G are both rotations?

Define an equivalence relation, and equivalence classes. Explain how an equivalence relation on a set S gives a decomposition of a S into disjoint subsets – a partition of S.

Define parallel lines as lines that coincide or do not intersect. Prove that "parallel" is an equivalence relation

State and prove the Pythagorean theorem.

State and prove SAS, ASA and SSS for triangles. Prove ASS or give a counterexample.

Let Λ be a reflection about a line l. What is its inverse?

Let $ABCD$ be a quadrilateral, and let P, Q, R and S be the midpoints of its four sides. Prove that $PQRS$ is a parallelogram.

Prove that the diagonals of a parallelogram bisect each other.

Prove that the sum of the angles of a triangle is a straight angle.

Define a dilation and prove that a dilation takes line segments into line segments and takes lines into themselves or into parallel lines. Prove that it also takes a convex set into a convex set.

Prove that a dilation maps an angle into an angle and preserves the degree of an angle.

Define alternate interior angles and opposite angles.

Define similarity and show that it is an equivalence relation.

Prove or disprove: All squares are similar. All triangles are similar. All rectangles are similar.

Prove that all triangles with two equal angles are similar.

Prove that if a triangle has sides of length a, b and c and $a^2 + b^2 = c^2$, then the angle opposite side c is a right angle.

State and prove Ceva's theorem for triangles.

Define a congruence and prove that every congruence is an isometry.

Prove the triangle inequality: the sum of the lengths of two sides of a triangle is strictly larger than the length of the third side.