Midterm 1, MAT 324, October 28, 2007

Answer each question on the paper provided. Write neatly and give complete answers. Each question is worth 10 points.

1. Define the outer measure of a set.
2. Prove every monotone function is measurable.
3. If f is measurable, show $|f|$ is measurable. Is the converse true? Prove or find give a counterexample.
4. State the characterization of Riemann integrable functions. Given an example of Lebesgue integrable function that is not Riemann integrable.
5. If E is measurable is it true that $m(E) = m(\overline{E})$, where $\overline{E}$ denote the closure of E? (the closure of a set E is the smallest closed set containing E).
6. If $E \subset [0, 1]$ is measurable, show that for any $\epsilon > 0$ there is a an closed set K so that $K \subset E$ and $m(E \setminus K) < \epsilon$.
7. Let $\{f_n\}$ be a sequence of measurable functions. Show that the set of x where $f_n(x)$ tends to $+\infty$ is measurable.
8. Suppose $f \geq 0$ is integrable and define $h_n = \min(f, n)$. Prove that $\int |f - h_n| dm \to 0$ as $n \to \infty$.
9. State the Dominated Convergence Theorem. Give an example of a uniformly bounded sequence of integrable functions where it does not apply, i.e., $\lim_n \int f_n dx \neq \int \lim_n f_n dx$.
10. If f is integrable, show $m(\{x : |f(x)| > \lambda\}) \leq \frac{1}{\lambda} \int |f| dx$.