PROBLEM SET 6

1. Let $f_n(x) = \sin(nx)$. Does the sequence f_n converge in $L^1[0, 2\pi]$?

2. Prove that if X is a metric space and p and q are distinct points of X, then there exit two disjoint subsets of X, one of which contains p and the other of which contains q.

3. Give an example of a topological space X whose topology does not come from any metric. That is, there is no metric on X whose open sets coincide with the open sets of the topological space X.

4. Give an example of a continuous function f such that the improper integral $\int_{-\infty}^{\infty} f(x)dx$ exists but f is not in $L^1(\mathbf{R})$. Explain why your example cannot be a positive function.

5. Let $f_n = n1_{[0,1/n]}$. Does there exist an integrable function g such that $f_n \le g$?