PROBLEM SET 5

1. Compute the integral of the Cantor-Lebesgue function $\int_0^1 F(x)dm$ from Chapter 2.

2. What is $\lim_{n\to\infty} \int_{-\infty}^{\infty} x^n e^{-n|x|} dm$? Find the limit and prove your answer.

3. Suppose $\{f_n\}$ is a sequence of functions that converges almost everywhere to a function f and define $F_n = \sup_{k=1,\dots,n} |f_n|$. Show that if the integrals of F_n remain bounded as $n \to \infty$ then $\lim_n \int f_n dm = \int f dm$.

4. Show that $\sum_{n=1}^{\infty} \cos^n(2^n x)$ converges for a.e. x, but diverges on a dense set of x's.

5. Let m be a measure defined on Borel sets in the reals $\mathbf{R}$ by:

$$m(E) = \int_E \frac{dx}{1+x^2}.$$

   Find $m(\mathbf{R})$.