Review for Midterm

The midterm will consist of questions from the following topics:

Well ordering principle and mathematical induction

Division algorithm

Greatest common divisor (GCD) and relatively prime numbers, Theorem 0.2,

Fundamental theorem of arithmetic, Modular arithmetic, $\mathbf{Z}_n$

Equivalence relations, equivalence classes and partitions of a set. Show that an equivalence relation on a set gives a partition of the set and visa versa.

Functions and their domains and ranges, injective, surjective and bijective functions

Definition of a group, dihedral groups and symmetries, $GL(2, \mathbf{R})$, Abelian or commutative groups

Finite groups and subgroups, the order of a group and the order of an element, the center of a group and the centralizer of an element

Cyclic groups and their generators and subgroups, $\mathbf{Z}_n$ and $U(n)$

Permutation groups and their cycles including transpositions, odd and even permutations and the alternating groups, symmetries of a square and tetrahedron as subgroups of permutation groups

Homomorphisms and isomorphisms of groups including automorphisms and inner automorphisms. Show that every group is a subgroup of a group of permutations.

Cosets and the Lagrange coset theorem using an equivalence relation

External and internal direct products, Show that $U(st) \cong U(s) \oplus U(t)$ if s and t are relatively prime. Prove that $U(105)$ is isomorphic to $U(144)$

Normal subgroups and factor groups, Classify all groups of order p^2 where p is prime. Conclude that they are all Abelian.

Let $\phi : G_1 \to G_2$ be a homomorphism. Define the kernel of ϕ and show that it is a normal subgroup of G_1. Prove that for any normal subgroup of a group, there is a homomorphism whose kernel is that subgroup.