Review for Midterm

Description of real and complex numbers as fields. Comnplex conjugate

Definition of a vector space over the real or complex numbers.

Subspaces, sums and direct sums

Span, linear independence, bases and dimension.

If a space has n linearly independent vectors, then a spanning set must have at least n elements. It follows that any two bases have the same number of elements.

Linear maps, their null spaces and ranges. The rank plus nullity theorem.

The matrix of a linear map. Matrix multiplication and composition of linear maps.

Invertible maps and their matrices. Injective and surjective maps.

Given a linear map $T : V \to W$, find bases of V and W that make the matrix of T as simple as possible. It should have r ones and the rest zeros, where r is the rank of the map.

Polynomials with real or complex coefficients. The degree of a polynomial. The division algorithm for polynomials. The fundamental theorem of algebra (You needn't know the proof.)

Factoring a polynomial with real coefficients into linear and quadratic factors

Eigenvalues and their eigenvectors.

Invariant subspaces of a linear map from a vector space into itself. A one-dimensional invariant space contains an eigenvector.

A set of eigenvectors with distinct eigenvalues is linearly independent. Therefore there cannot be more eigenvalues than the dimension of the space.

If $T \in L(V)$, and $p(x)$ is a polynomial, define $p(T)$.

If V is a finite dimensional complex vector space of positive dimension, and $T \in L(V)$, then T has an eigenvalue. Also there is a basis of V for which the matrix of T has upper-triangular form. Also the diagonal elements of this matrix are the eigenvalues of T.

If T is in $L(V)$ and V is given a basis, then with respect to this basis, there is a matrix for T. How does the matrix change if one changes the basis?

If T is as above, and T has n distinct eigenvalues where n is the dimension of V, then one can choose a basis of V consisting of eigenvectors of T, and the matrix of V with respect to this basis has diagonal form.

If V is a real finite positive dimensional vector space, and $T \in L(V)$, then T must have either an eigenvector or a two-dimensional invariant supspace.

Give an example of such a T which has no eigenvalue or eigenvector.

Projections onto and along subspaces.

If V is real as above and odd-dimensional, and T is as above, then T has an eigenvector.

Definition of Inner Product for real and complex vector spaces. Dot product as an example.

Norm defined by inner product. $< v, w >= 0$ implies $||u||^2 + ||v||^2 = ||u+v||^2$

Prove the Schwartz inequality and triangle inequality.