10th Assignment

1. Find the (possibly complex) eigenvalues, eigenvectors and singular values of the matrix:

$$\begin{pmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$$

2. Find an isometry and a positive operator whose product is the operator given by the matrix above.

3. Prove that the trace of a non-zero positive operator is positive. Give an example of a self-adjoint operator with positive trace that is not positive.

4. Prove that matrix multiplication is associative. You may use the fact that with appropriate bases, the matrix of the composition of two linear transformations is the product of the matrices of the transformations

5. Find a two by two matrix of complex numbers that does not have a square root. Can such a matrix have two distinct eigenvalues?

6. Let V be an n-dimensional vector space with basis $\{e_1, e_2, ..., e_n\}$. Define $T : V \to V$ by $T(e_i) = e_{i+1}$ for $i = 1...n-1$ and $T(e_n) = 0$. Prove that $T^n = 0$. Prove that T does not have a square root.