Review for MAT 341 Final
May, 2016

Solve first order linear ode's both homogeneous and non-homogeneous
Solve $\ddot{u} + k\dot{u} + pu = 0$ for k and p constants.
Show that there is only one solution u of the equation above with a given $u(0)$ and $\dot{u}(0)$.
Understand superposition for linear equations. (the sum of two solutions is also a solution and a constant times a solution is a solution)
Boundary value problem: Solve $\ddot{u} + k\dot{u} + pu = 0$ given $u(a)$ and $u(b)$. Is the solution necessarily unique?
Find heat flow in a cylinder and the velocity of water in a pipe using the assumptions made in class.
Define a periodic function.
Define the Fourier series of a function on the interval $(-a, a)$. This includes the formula for the Fourier coefficients.
Define orthogonal and orthonormal sets in a vector space with inner product.
Define the span of a set of vectors.
Define even and odd functions.
Prove that $\{\sin nx\}$ is an orthogonal set with respect to the inner product:
$\langle f, g\rangle = \int_{-\pi}^{\pi} f(x)g(x)dx$
Define pointwise and uniform convergence for a sequence of functions. Show that the uniform limit of a sequence of continuous functions is continuous.
Let $\{a_n, b_n\}$ be the Fourier coefficients of a function f. Show that if

$$\Sigma_0^\infty |a_n| + |b_n| < \infty$$

then the Fourier series of f converges uniformly.
Define convergence in mean.
Given $f(x) = \Sigma_1^\infty a_n \cos nx$ find a solution of

$$\ddot{u} + \alpha\dot{u} + \beta u = f$$

assuming that α is not zero.
Understand the heat equation for the temperature in a uniform rod as a function of one space variable and time. Understand both transient and steady-state solutions
Solve the heat equation using Fourier series in the case of insulated ends and in the case of fixed temperature at the ends
Show that

$$\partial_t u = \partial_x^2 u$$

$u(x, t) = f(x)$ has a unique solution assuming that $0 < x < 1$ and assuming that $u(0, t) = a$ and $u(1, t) = b$.
Understand that the solution of the wave equation

$$\partial_x^2 u = \frac{1}{c^2}\partial_t^2 u$$

with zero boundary data approximates the motion of a string with fixed ends.
Show that for any differentiable functions ψ and ϕ, $u(x,t) = \psi(x+ct)+\phi(x-ct)$ solves the wave equation above.
Understand Laplace's equation (the potential equation) in rectangular, polar, cylindrical and spherical coordinates
Same for the wave and heat equations
Derive the potential equation in polar coordinates
Use the separation of variables technique together with sequences of orthogonal functions to construct solutions to all of these equations.
The sequences should include Fourier series, Bessel funtions and or Legendre polynomials as appropriate to each problem.
Find a function on a square with given values at the corners which satisfies the potential equation
Solve $\Delta u = H$ where H is a constant. Is the solution unique?
Let Ω be a bounded domain in the plane. Assume that

$$\Delta u = \lambda^2 u$$

with u equal to zero on the boundary of Ω. Prove that u is zero.
Define spherical coordinates and know the range of each variable; write the transformation from spherical to rectangular coordinates.
Define the Laplace transform and compute the transforms of e^{kt} and of polynomials. Also compute the transforms of sine, cosine, sinh and cosh.