Problem 1. (a) Is the function \( f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \) with formula \( f(x) = 1/x \) bounded?
(b) Is \( f \) bounded when restricted to the domain \( D := [1, \infty) \)?
(c) Find the supremum and infimum of \( f(D) \) or explain why they do not exist.

Problem 2. State the Archimedean property for rational numbers and prove that it holds.
Note: Do not use the Completeness Property of the real numbers, i.e. do not imitate the proof of 2.4.3. There is a different short argument that applies when \( x \) is rational.

Problem 3. Let \( S \subset \mathbb{R} \) be nonempty. Show that \( u \in \mathbb{R} \) is an upper bound for \( S \) if and only if the conditions \( t \in \mathbb{R} \) and \( t > u \) imply that \( t \notin S \).

Problem 4. (a) Use induction to prove that a nonempty finite subset of \( \mathbb{R} \) contains its supremum.
Hint: Argue by induction on the number of elements in the finite set.
(b) Give an example to show that this statement does not hold for every infinite subset of \( \mathbb{R} \).

Problem 5. Prove Theorem 2.5.1 case (iii).