Problem 1. Prove the identity: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Problem 2. (i) Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$. Is it true that $f(A \cap B) = f(A) \cap f(B)$ for all subsets $A, B$ of $\mathbb{R}$? Give a brief proof or a counterexample.
(ii) Do the same question using the function $h : \mathbb{R} \to \mathbb{R}$ where $h(x) = x^3$.

Problem 3. Let $f : A \to B$ be a function and suppose that $C \subseteq A$ and $D \subseteq B$. Are the following statements true or false? Justify your answers by a brief proof or a counterexample.
(i) $f(A \setminus C) \subseteq f(A) \setminus f(C)$.
(ii) $f^{-1}(B \setminus D) = f^{-1}(B) \setminus f^{-1}(D)$.

Hint: as in question 2, try some examples. You can try functions $f : \mathbb{R} \to \mathbb{R}$ or you can try functions $f : A \to B$ where $A$ and $B$ are finite sets.

Problem 4. Suppose that $f : A \to B$ and $g : B \to C$ are functions such that the composite $g \circ f$ is injective. Is $f$ necessarily injective? What about $g$? Give brief proofs or counterexamples.

Problem 5. Prove by mathematical induction: $2^n < n!$ for all $n \geq 4$. \(\text{Note: } n! = n(n - 1)(n - 2) \cdots 3 \cdot 2 \cdot 1.\)