

## Math 319/320 Worksheet 3

Name:

ID:

**Problem 1.** (i) Construct a function  $f : \mathbb{N} \rightarrow \mathbb{N}$  that is injective but not surjective.

(ii) Show that if  $S$  is an infinite set then there is a function  $f : S \rightarrow S$  that is injective but not surjective. You may use the fact that  $S$  is infinite if and only if there is an injection  $\mathbb{N} \rightarrow S$ .

**Problem 2.** (i) Consider the set  $\mathbb{N} \times \mathbb{N}$  of all ordered pairs  $(p, q)$  of integers. Show that it is denumerable, i.e. show how to construct a bijection  $\mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$ .

(ii) Show that if  $A$  and  $B$  are denumerable, disjoint sets, then  $A \cup B$  is denumerable.

**Problem 3.** Let  $a, b, c, d$  be real numbers which satisfy  $0 < a < b < c < d$ .

a) Is it true that  $bc < ad$ ? If it is true prove the inequality. If it is not true give an example of four real numbers which violate the inequality.

b) Is it true that  $ca < bd$ ? If it is true prove the inequality. If it is not true give an example of four real numbers which violate the inequality.

c) Assume that  $0 < c^2 < c$  for some real number  $c$ . Show that  $0 < c < 1$ .

**Problem 4.** a) Show that if  $x$  and  $y$  are rational numbers then  $x + y$  and  $xy$  are rational numbers.

b) If  $x \neq 0$  is rational and  $y$  irrational, show that  $xy$  is irrational.

c) If  $x$  and  $y$  are irrational, is it always true that  $x + y$  is irrational? Explain.