Math 319 Homework 9
Due Tuesday, November 8, 2005

Problem 1. 1: (i) Let $f : (0, 5) \to \mathbb{R}$ be the function $f(x) = 2x^2 + 3$. Prove from the definition that $\lim_{x \to 2} f = 11$.

(ii) Find $M, \delta > 0$ so that $|f(x)| \leq M$ for all $x \in A \cap (2 - \delta, 2 + \delta)$.

Problem 2 Let $c$ be a cluster point of $A$ and $f : A \to \mathbb{R}$. Suppose that $\lim_{x \to c} f = L$. Show that there is a $\delta > 0$ such that $f$ is bounded on the set $A \cap (c - \delta, c + \delta)$.

Note: Part (ii) of Problem 1 is an example of this general statement.

Problem 3: Let $A = \{1/n : n \in \mathbb{N}\}$ and let $f : A \to \mathbb{R}$ be the function $f(x) = 1/(1 + x)$.

(i) Write down the values of $f$ at the points $x = 1, 1/2, 1/3$.

(ii) Does $\lim_{x \to 0} f$ exist? If so evaluate it.

(iii) Does $\lim_{x \to 1/3} f$ exist? If so evaluate it.

Justify your answers.