

## Math 319 Homework 9

Due Tuesday, November 8, 2005

**Problem 1. 1:** (i) Let  $f : (0, 5) \rightarrow \mathbb{R}$  be the function  $f(x) = 2x^2 + 3$ . Prove from the definition that  $\lim_{x \rightarrow 2} f = 11$ .

(ii) Find  $M, \delta > 0$  so that  $|f(x)| \leq M$  for all  $x \in A \cap (2 - \delta, 2 + \delta)$ .

**Problem 2** Let  $c$  be a cluster point of  $A$  and  $f : A \rightarrow \mathbb{R}$ . Suppose that  $\lim_{x \rightarrow c} f = L$ . Show that there is a  $\delta > 0$  such that  $f$  is bounded on the set  $A \cap (c - \delta, c + \delta)$ .

Note: Part (ii) of Problem 1 is an example of this general statement.

**Problem 3:** Let  $A = \{1/n : n \in \mathbb{N}\}$  and let  $f : A \rightarrow \mathbb{R}$  be the function  $f(x) = 1/(1 + x)$ .

(i) Write down the values of  $f$  at the points  $x = 1, 1/2, 1/3$ .

(ii) Does  $\lim_{x \rightarrow 0} f$  exist? If so evaluate it.

(iii) Does  $\lim_{x \rightarrow 1/3} f$  exist? If so evaluate it.

Justify your answers.