Math 319/320 Homework 1

Due Thursday, September 8, 2005

Problem 1. Prove the identity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Problem 2. Consider the function $f : \mathbb{R} \setminus \{1\} \to \mathbb{R}$ given by $f(x) = x^2/(x-1)$.

(i) Graph it.

(ii) Find $f(A)$ where A is the interval $(1.5, 4)$.

(iii) Find $f^{-1}(B)$ where $B = [1, 4]$.

(iv) Find two subsets C, D of $\mathbb{R} \setminus \{1\}$ such that $f(C) \cap f(D) \neq f(C \cap D)$.

Problem 3. Let $f : A \to B$ be a function and suppose that $C \subseteq A$ and $D \subseteq B$. Are the following statements true or false (for every choice of f, C, D)? Justify your answers by a brief proof or a counterexample.

(i) $f(A \setminus C) \subseteq f(A) \setminus f(C)$.

(ii) $f^{-1}(B \setminus D) = f^{-1}(B) \setminus f^{-1}(D)$.

Hint: as in question 2, try some examples. You can try functions $f : \mathbb{R} \to \mathbb{R}$ or you can try functions $f : A \to B$ where A and B are finite sets.

Problem 4. Suppose that $f : A \to B$ and $g : B \to C$ are functions such that the composite $g \circ f$ is surjective. Is g necessarily surjective? What about f? Give brief proofs or counterexamples.

Problem 5. Prove by mathematical induction: $3^{2n} - 1$ is divisible by 8 for all $n \geq 1$.