Problem 1. (i) Consider the statement:
\[ 2 + 5 + \cdots + 3n - 1 = \frac{n(3n + 1)}{2}. \]
What does it say when \( n = 1, n = 2, \) and \( n = 3? \)
(ii) Use mathematical induction to prove it for all \( n. \)

Problem 2. Use mathematical induction to show that \( n^3 + 3n^2 - n \) is divisible by 3 for all \( n \geq 1. \)

Problem 3. Prove ONE of the following results:
EITHER: Let \( A = (0,2) \) and suppose that \( c \) is a cluster point of the complement \( \mathbb{R} \setminus A. \) Prove that \( c \notin A. \)
OR: (ii) Let \( f : A \to \mathbb{R} \) be any function and let \( s := \sup\{f(x) : x \in A\}. \) Show that there is a sequence \( x_n \in A \) such that \( \lim f(x_n) = s. \)

Problem 4. (i) Let \( x > 0. \) Prove that \( \lim_{n \to \infty} \frac{1}{1+nx} = 0. \)
(ii) Let \( 0 < b < 1. \) Prove that \( \lim b^n = 0. \)

(Use Archimedes Principle and Bernoulli’s inequality, both of which are now on the sheet)

Problem 5. Suppose that \( (x_n), (y_n) \) are sequences such that \( \lim x_n = 2 \) and \( \lim y_n = 3. \)
Prove from the definitions that there is an integer \( K \) such that \( x_n < y_n \) for all \( n \geq K. \)

Problem 6. Let \( f : \mathbb{R} \to \mathbb{R} \) be the function \( f(x) = \frac{3x}{x^2 + 2}. \) Prove from the definition that \( \lim_{x \to 1} f(x) = 1. \)

Problem 7. Describe examples satisfying the following conditions. Justify your answers.
(i) A bounded set \( A \) and a continuous function \( f : A \to \mathbb{R} \) that is not bounded.
(ii) A convergent sequence that is not monotonic.
(iii) A countable set that is bounded above but does NOT contain its supremum.

Problem 8. Define the sequence \( (x_n) \) recursively by setting \( x_1 = 1 \) and \( x_n = 1 + \frac{x_{n-1}}{2}. \)
(i) Show that \( (x_n) \) is monotonic increasing.
(ii) Show that it converges and find its limit.
Problem 9. Which of the following sequences are convergent? Justify your answers.

(i) \( x_n = \frac{(-1)^n \sin n}{n + 1} \);  
(ii) \( x_n = \frac{(-2)^n}{n + 1} \);  
(iii) \( x_n = \frac{n - 3}{2n - 1} \).