Problem 1. (15 points) Prove ONE of the following statements.
EITHER: (i) Prove that a convergent sequence is bounded.
OR: (ii) Prove that if $c$ is a cluster point of $A$ there is a sequence $(a_n)$ with limit $c$
 such that $a_n \in A \setminus \{c\}$ for all $n$.
OR: (iii) If $\lim x_n = x$ and $\lim y_n = y$ then $\lim x_ny_n = xy$. 

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Answer all the following questions, justifying all your statements. If you need more space, please write on the back of the sheets. There are four questions, worth a total of 50 points. Good luck!
Problem 2. (10 points) Define the sequence \((x_n)\) inductively by setting \(x_{n+1} = \frac{x_n}{2} - 1\).

(i) Show that \((x_n)\) is monotone increasing if \(x_1 \leq -2\) and is monotone decreasing if \(x_1 \geq -2\).

(ii) Suppose that \(x_1 = 0\). Does the sequence \((x_n)\) have a limit? If so, what is it?

Problem 3. (10 points) Let \(f : \mathbb{R} \to \mathbb{R}\) be the function \(f(x) = 2x + x^2\). Prove from the definition that \(\lim_{x \to 1} f(x) = 3\).
Problem 4. (15 points) Which of the following sequences \((x_n)\) are convergent? If they are convergent, what are their limits? Prove your claims.

(i) \(x_n = \frac{(-1)^n n}{n + 3}\),   
(ii) \(x_n = \frac{(-1)^n n}{n^2 + 3}\),   
(iii) \(x_n = \frac{n^2}{n + 3}\). 