Math 319 Second Midterm

November 10, 2005

Name: **School ID:**

Answer all the following questions, justifying all your statements. If you need more space, please write on the back of the sheets. There are four questions, worth a total of 50 points. Good luck!

Problem 1. (15 points) Prove ONE of the following statements.
EITHER: (i) Prove that a convergent sequence is bounded.

OR: (ii) Prove that if c is a cluster point of A there is a sequence (a_n) with limit c such that $a_n \in A \setminus \{c\}$ for all n.

OR: (iii) If $\lim x_n = x$ and $\lim y_n = y$ then $\lim x_n y_n = xy$.

1	
2	
3	
4	
Total	

Problem 2. (10 points) Define the sequence (x_n) inductively by setting $x_{n+1} = x_n/2 - 1$.
(i) Show that (x_n) is monotone increasing if $x_1 \leq -2$ and is monotone decreasing if $x_1 \geq -2$.

(ii) Suppose that $x_1 = 0$. Does the sequence (x_n) have a limit? If so, what is it?

Problem 3. (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = 2x + x^2$. Prove from the definition that $\lim_{x \to 1} f(x) = 3$.

Problem 4. (15 points) Which of the following sequences (x_n) are convergent? If they are convergent, what are their limits? Prove your claims.

$$\text{(i)} \qquad x_n = \frac{(-1)^n n}{n+3}, \qquad \text{(ii)} \qquad x_n = \frac{(-1)^n n}{n^2+3}, \qquad \text{(iii)} \qquad x_n = \frac{n^2}{n+3}.$$