Project 1
(due by 12/14/06 – 5:00pm)

Discuss the problem below in a concise and precise essay, at most 5 typed pages long. Whenever you use a reference, quote it and do not copy. Use your own words.

Consider the linear operator $T : \mathbb{R}^4 \to \mathbb{R}^4$ which has the matrix

$$
\begin{bmatrix}
-4 & 3 & 1 & -1 \\
-6 & 5 & 0 & 0 \\
0 & 0 & 0 & 4 \\
0 & 0 & -1 & 4
\end{bmatrix}
$$

with respect to the standard basis.

1. Argue that there is a basis $v_1, v_2, v_3, v_4$ of $\mathbb{R}^4$ for which the matrix of $T$ is upper-triangular by explicit construction, using the methods of Chapter 5. In particular, all eigenvalues are real. Give them and compute the matrix.

2. Find all invariant subspaces of $T$. Why is there no basis of eigenvectors, so $T$ is not diagonalizable?

3. Finally, apply the Gram-Schmidt process to the above basis and construct an orthonormal basis $e_1, e_2, e_3, e_4$ and the corresponding upper-triangular matrix for $T$ according to Schur’s Theorem (Corollary (6.27) in the text book.).