Math 171 - Fall 2015
Practice test for Final Examination

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1. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \frac{|\sin x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} .$$

Let

$$F(x) = \int_0^x f(t)dt.$$

(a) Show that F continuous.

(b) At which points of $\mathbb{R}$ is F differentiable?

(c) If $F'(x)$ exists, is F' continuous at x?

2. Let $f : [-\pi/2, \pi/2] \to \mathbb{R}$ be defined by

$$f(x) = e^{-x}(\sin x)^2.$$

Find the points where f attains its absolute maximum and absolute minimum.

3. Let

$$f(x) = \int_0^x t\cos(\tfrac{1}{t})dt, \qquad x \in \mathbb{R}.$$

Find all points $x \in \mathbb{R}$ where f is differentiable, and compute $f'(x)$ at those points.

4. Consider the function

$$f(x) = \frac{1}{2}x^2, \quad -1 \leq x \leq 1.$$

Find the arclength of its graph.

5. Compute the following integrals.

(i)

$$\int_1^4 x \ln x dx.$$

(ii)

$$\int_0^2 \frac{dx}{(x^3 + 4x + 5)^2}$$

6. Decide with proof if the improper integral

$$\int_0^\infty \frac{(\sin(x))^2 \ln x}{x^2} dx$$

converges or diverges.

7. Find the general solution of the differential equation

$$x\frac{dy}{dx} - (1+x)\tan y = 0.$$

8. Decide, with explanation, whether the following series converge or diverge.

(a)

$$\sum_{n=2}^\infty \frac{\ln n}{n^2}.$$

(b)

$$\sum_{n=2}^\infty e^{-n}(\ln n)^4.$$

(c)

$$\sum_{n=0}^\infty \frac{1}{2+n\sin(\frac{\pi}{2}n)}.$$

9. Find the tailor series about 0 for the function

$$f(x) = \ln(1+x^2).$$

10. Find the interval of convergence of the power series

$$\sum_{n=0}^\infty e^n(x-5)^n.$$