

**Math 171 - Fall 2015**  
**First Practice Examination**

Instructor: **Dror Varolin**

1. Does the function  $f(x) = x^3 - \cos x$  have a root?
2. Are there any values of  $c$  for which the function

$$f_c(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ c & x = 0 \end{cases}$$

is differentiable? Justify your answer using the definition of the derivative.

3. Determine the set of points  $x > 0$  where the function  $f(x) = x + 2 \cos x$  is increasing.
4. Compute the following limits.

(a)

$$\lim_{t \rightarrow 0} \frac{1 - \cos t}{t^2}.$$

(b)

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 + 6x + 8}.$$

5. Compute the derivative of the function

$$F(x) = \int_0^{x^3} t \sin(x - t) dt.$$

6. Compute the following limits:

(a)

$$\lim_{x \rightarrow 2} \frac{(|x| - 2) \sin(\pi x)}{1 - \cos(\pi x)}.$$

(b)

$$\lim_{x \rightarrow 2} \frac{1}{x - 2} - \frac{1}{\sin(\pi x)}.$$