

Math 171 - Fall 2015
Second Practice Examination

Instructor: **Dror Varolin**

1. Sketch the graph of the function

$$f(x) = xe^{-x^2}, \quad x \in \mathbb{R}.$$

2. Let

$$f(x) := \int_0^{x^2} e^{-t^2/2} dt, \quad x > 0$$

and define

$$a := \int_0^{\infty} e^{-t^2/2} dt.$$

(a) Prove that a is finite.

(b) Prove the $f : (0, \infty) \rightarrow (0, a)$ is invertible.

3. Compute the following limits

$$(a) \lim_{x \rightarrow \infty} \frac{\ln x^2}{x^2} \quad (b) \lim_{x \rightarrow 0} \frac{x^3}{x - \sin x} \quad (c) \lim_{x \rightarrow 0} \frac{8\sqrt{1+x} - (4x - x^2)}{x^3}.$$

4. Compute the following integrals.

$$(a) \int_1^5 6x^3 e^{-x^2} dx \quad (b) \int_3^8 \frac{dt}{t^3 \sqrt{t^2 - 4}}.$$

5. Prove that the integral

$$\int_0^{\infty} \frac{(\ln x)^4}{1+x^2} dx$$

converges.

6. Find the general solution of the differential equation

$$\dot{y} = -2ty + 2t \cos(t^2)$$