(1) (a) Let \( f : \mathbb{D} - \{0\} \rightarrow \mathbb{C} \) be a holomorphic function. Show that if \( f \) has an essential singularity at 0, then \( f \) is not injective. 
   Hint: Use the Weierstrass-Casorati and Open Mapping Theorems.
(b) Let \( f : \mathbb{C} \rightarrow \mathbb{C} \) be an injective holomorphic function. Show that \( f \) is affine linear, i.e., that 
   \[ f(z) = az + b \]
   for some \( a, b \in \mathbb{C} \) with \( a \neq 0 \).
(2) Prove Theorem 4.4.35; the theorem on normal forms of meromorphic functions.
(3) Prove Theorem 4.5.16; the general version of the residue Theorem.
(4) Let \( g \in \mathcal{C}_\infty(\mathbb{C}) \). Consider the inhomogeneous Cauchy-Riemann Equations
   \[ \frac{\partial u}{\partial \bar{z}} = g \]
   for the unknown function \( u \).
   (a) Show there exists some \( u \) satisfying the equation (CR) whose support is not compact.
   (b) Show that if there exists a solution \( u \) of (CR) with compact support then
   \[ \int_{\mathbb{C}} z^n g(z) dA(z) = 0 \quad \text{for all } n = 1, 2, \ldots . \]
   (c) Show that if
   \[ \int_{\mathbb{C}} z^n g(z) dA(z) = 0 \quad \text{for all } n = 1, 2, \ldots . \]
   then the solution
   \[ u(z) := -\frac{1}{\pi} \int_{\mathbb{C}} \frac{g(\zeta) dA(\zeta)}{\zeta - z} \]
   has compact support, and in fact its support is contained in the union of the support of \( g \) and every bounded component of the complement of the support of \( g \).
(5) Consider the domain
   \[ \Omega := \{ z \in \mathbb{C} ; 1 < |z| < 2 \text{ or } 4 < |z| < 5 \} . \]
   (a) Let \( f \in \mathcal{O}(\Omega) \), and let \( K \subset \subset \Omega \). Show that there is a holomorphic function \( g \in \mathcal{O}(\mathbb{C} - \{0, 3\}) \) such that
   \[ \sup_K |f - g| < 1/2 . \]
   (b) Let
   \[ L := \{ z \in \mathbb{C} ; 4/3 \leq |z| \leq 3/2 \text{ or } 4.1 \leq |z| \leq 4.5 \} \subset \subset \Omega . \]
   Find a function \( h \in \mathcal{O}(\Omega) \) such that there is no holomorphic function \( g : \mathbb{C} - \{0\} \rightarrow \mathbb{C} \) satisfying
   \[ \sup_L |g - h| \leq 1 . \]
(6) Show that there is a holomorphic function \( f : \mathbb{C} - \{0\} \rightarrow \mathbb{C} - \{0\} \) such that
   \[ f(1/n) = 1 \quad \text{and} \quad f'(1/n) = 1/\sqrt{n} \quad \text{for all } n = 1, 2, 3, \ldots . \]
What kind of singularity can \( f \) have at the origin?