(1) Find all $2 \times 2$ matrices $A$ with real coefficients with positive determinant such that $|Av| = |v|$ for all vectors $v \in \mathbb{R}^2$ (thought of as $2 \times 1$ matrices). The norm is the usual norm in $\mathbb{R}^2$, i.e., $| \begin{pmatrix} a \\ b \end{pmatrix} | = \sqrt{a^2 + b^2}.$

(2) Prove that for $a$ and $b$ in the unit disk,
\[
\left| \frac{b - a}{1 - ba} \right| < 1,
\]
and that if $|a| = 1 > |b|$ or $|b| = 1 > |a|$, then
\[
\left| \frac{b - a}{1 - ba} \right| = 1.
\]

(3) Find sequences $\{a_j\}, \{b_j\}$ such that $|a_j| \to 1, |b_j| \to 1$ and
\[
\left| \frac{b_j - a_j}{1 - b_ja_j} \right| \not\to 1.
\]
What are all the possible limits of such sequences?

(4) Prove Lagrange’s Identity
\[
\left| \sum_{j=1}^{n} a_jb_j \right|^2 = \sum_{j=1}^{n} |a_j|^2 \sum_{j=1}^{n} |b_j|^2 - \sum_{1 \leq j < k \leq n} |a_jb_k - a_kb_j|^2.
\]

(5) Show that the function
\[
f(z) := \begin{cases} 
\frac{z^2}{z}, & z \neq 0 \\
0, & z = 0
\end{cases}
\]
satisfies the Cauchy-Riemann equations at the origin, but is not differentiable there.

(6) Let $\Omega$ be an open connected subset of $\mathbb{R}^2$ containing the origin. Show that the function
\[
u(x, y) = \log(x^2 + y^2)
\]
is harmonic in $\Omega - \{0\}$. Prove that $u$ is not the real part of a holomorphic function in $\Omega - \{0\}$. 