1. Does the function $f(x) = x^3 - \cos x$ have a root?
2. Are there any values of $c$ for which the function

$$f_c(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ c & x = 0 \end{cases}$$

is differentiable? Justify your answer using the definition of the derivative.
3. Determine the set of points $x > 0$ where the function $f(x) = x + 2 \cos x$ is increasing.
4. Compute the following limits.

(a) $\lim_{t \to 0} \frac{1 - \cos t}{t^2}$.

(b) $\lim_{x \to -2} \frac{x^3 + 8}{x^2 + 6x + 8}$.

5. Compute the derivative of the function

$$F(x) = \int_0^x t \sin(x-t)dt.$$ 

6. Compute the following limits:

(a) $\lim_{x \to 2} \frac{(|x| - 2) \sin(\pi x)}{1 - \cos(\pi x)}$.

(b) $\lim_{x \to 2} \frac{1}{x-2} - \frac{1}{\sin(\pi x)}$. 
