MAT 342 Midterm

Applied Complex Analysis

March 28, 2019

Name: ____

Student ID: ______________________________________

Instructions:

There are 6 problems on 6 pages (plus this cover sheet) in this exam. Make sure that you have them all.

Answer all questions below. You may use the backs of pages, but in this case indicate where your work for each problem is located. You must show all of your work and provide **complete justifications** for all of your claims. Insufficient justifications will not receive full credit.

Leave all answers in exact form (that is, do not approximate π, square roots, and so on.)

You may **not** use books, notes, calculators, cell phones or other electronic devices.

You have 1 hour and 10 minutes to complete this exam.

Problem	1	2	3	4	5	6	Total
Points	10	10	10	10	10	10	60
Score							

Name: ________________________ **Student ID:** ________________________

[10 pts] **Problem 1.**

(i) When is a function called entire?

(ii) What is the largest domain in which the function

$$f(z) = \frac{\mathrm{Log}(-iz)}{z^2 + i}$$

is analytic and why is it analytic there? What are the singular points of f? Explain carefully.

[10 pts] **Problem 2.**

(i) What does it mean for a function $f\colon S \to \mathbb{C}$ to be differentiable at a point $z_0 \in S$, where S contains a neighborhood of z_0?

(ii) Examine whether the function $f(z) = \overline{z}\,\operatorname{Im} z$ is differentiable at any point z, in which case, compute $f'(z)$. Is f analytic at any point of $\mathbb{C}$?

[10 pts] **Problem 3.** Consider the domain D consisting of the complex plane with the positive imaginary axis and 0 removed.

(i) Describe the branch of the power function $f(z) = z^i$ defined in the domain D for which $f(1) = e^{-2\pi}$.

(ii) Calculate $f(1+i)$.

[10 pts] **Problem 4.**

(i) State the Cauchy-Goursat theorem.

(ii) Calculate the contour integral

$$\int_C (e^{-\sin z} + \overline{z}\,\mathrm{Re}\,z)dz,$$

where C is the contour consisting of the curve $z(t) = t^2 + it^3$, $0 \leq t \leq 1$, followed by the line segment from $1 + i$ to 0.

[10 pts] **Problem 5.** Consider the set $S = \{z : 0 \leq \operatorname{Re} z < 1\}$.

(i) Is the set S open, closed, or neither? Explain carefully.

(ii) Find the image of the set S under the function $f(z) = e^z$. As part of your answer, include a sketch of $f(S)$, using solid lines to indicate boundaries which are included and dashed lines to indicate boundaries not included. Shade the interior of $f(S)$ (if there is one).

[10 pts] **Problem 6.**

(i) Is the function $u(x, y) = x^2 e^{y^2}$ the real part of an analytic function? Explain carefully.

(ii) Suppose that g is an entire function such that $g(z) = e^z$ for all points z lying in the unit disk $|z| < 1$. What can you say about the function g?