

## Homework 5 (due 3/7)

MAT 342: Applied Complex Analysis

Read Sections 30–40 from Chapter 3.

Problems from the textbook:

§30: 3, 4, 8, 10

§33: 3, 4

§38: 15

§40: 3

Additional problems to hand in:

**Problem 1.** Write  $\operatorname{Re}(e^{1/z^2})$  in terms of  $x$  and  $y$ . Explain why this function is harmonic everywhere except at the origin.

**Problem 2.** (i) Consider the function  $g(z) = 2z - 5i$ . What is the *inverse image* of the set  $S = \{x + iy : x \leq 0\}$  under this function? (That is, find the set of points  $z$  such that  $g(z) \in S$ .)

(ii) Consider the function

$$f(z) = \frac{\operatorname{Log}(2z - 5i)}{z^4 + i}.$$

What is the largest domain on which this function is analytic? Justify your claims.

(iii) What are the singular points of the function  $f$ ?

**Problem 3.** Explain, without computing derivatives and showing that Laplace's equation holds, why the function  $u(z) = \log |z|$  is harmonic everywhere except at the origin.