Dr. Simon Albrecht *Spring 2016*

Name: ______________________________ **ID:** ____________________

MAT 342 Applied Complex Analysis
Final Exam Example

May 2016

1. (12 pts, 4 pts each)

   a) Define the notion *complex differentiable.*

   b) Define the *principle branch of the logarithm.*

   c) State *Cauchy's residue theorem.*

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2. (12 pts, 4 pts each)

   a) Find the multiplicative inverse of $3 + 4i$ and write the solution in rectangular form.

   b) Find all $z \in \mathbb{C}$ such that $z^2 = 4i$.

   c) Prove the triangle inequality: For all $z, w \in \mathbb{C}$, the inequality

   $$|z + w| \leq |z| + |w|$$

   holds.

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3. (10 pts) Find all $z \in \mathbb{C}$ such that

$$z^4 + z^3 + z^2 + z + 1 = 0.$$

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4. (12 pts) Let f be an entire function such that

$$f(z) = f(z+1) = f(z+i)$$

for all $z \in \mathbb{C}$. Prove that f is constant.

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5. (10 pts) Let p be a polynomial of degree d_p and let q be a polynomial of degree d_q with $\max\{d_p, d_q\} \geq 1$. Assume that q is not constantly 0 and that p and q do not share a common zero. Let $f : \mathbb{C} \setminus \{z \in \mathbb{C} \mid q(z) = 0\} \to \mathbb{C}$ be given by

$$f(z) = \frac{p(z)}{q(z)}.$$

Let $z_0 \in \mathbb{C}$. Prove that there exists some $z \in \mathbb{C}$ such that $f(z) = z_0$.

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6. (12 pts) Find the Laurent series of

$$f(z) = \frac{1}{(z-1)(z-3)}$$

in $\{z \in \mathbb{C} \mid 0 < |z-1| < 2\}$.

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7. (12 pts, 4 pts each) Let

$$f(z) = \frac{1}{(z-2)(z-4)}.$$

Find the contour integrals of f along the circles about the origin of radius 1, 3 and 5, taken in counterclockwise direction.

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8. (20 pts, 10 pts each) Compute both

   a)
   $$\int_0^\infty \frac{1}{1+x^4}dx \quad \text{and}$$

   b)
   $$\int_{-\infty}^\infty \frac{x\sin(ax)}{x^4+4}dx \quad \text{where } a > 0$$

   using residues.