MAT 342 FALL 2014 FINAL EXAM

NAME : ID :

THERE ARE NINE (9) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		50pts
2		50pts
3		50pts
4		50pts
5		50pts
6		50pts
7		50pts
8(a,b)		50pts
9(a,b)		50pts
Total		450pts
8(c)		10pts
9(c)		10pts

!!! WRITE YOUR NAME, STUDENT ID. BELOW !!!

NAME : ID :

1. (50pts) (a): Find complex numbers z such that $z^2 = i$.

(b): Solve for z such that $\sin(z) = 2i$.

2. (50pts) (a): What's the image D_2 of the region $D_1 = \{z \in \mathbb{C}; \pi/2 < \mathrm{Re}(z) < \pi\}$ under the map $w = iz$?
(b): What's the image of the region D_2 (from above) under the map $w = e^z$?

3. (50pts) For each of the following two functions $v(z)$. Determine whether $v(z)$ can be the **imaginary** part of an analytic function $f(z)$. If yes, then find what $f(z)$ is.
(a): $v(z) = x^2 - y^2$.
(b): $v(z) = x^2 + y^2$.

4. (50pts) Calculate the following contour integrals.
(a):

$$\int_{|z-\pi|=3} \frac{\sin(z)}{(z-\pi)^4} dz.$$

(b):

$$\int_0^i \frac{1}{z+1} dz$$

along the straight line from 0 to i.

5. (50pts) Denote $f(z) = \text{Log}(1+z)$.

(a): Integrate an appropriate geometric series to get the Taylor series for $f(z)$ centered at 0 What's the radius of convergence?

(b): Find the Taylor series of $f(z)$ centered at 1. What's the radius of convergence?

6. (50pts) Find the Laurent series centered at 0 of the following function in two different regions.

$$\frac{1}{(z-1)(z-2)}$$

(a). $|z| < 1$. **(b).** $1 < |z| < 2$.

7. (50pts) Calculate the contour integrals using residues:
(a):

$$\int_{|z|=3} \frac{dz}{(z-3)^2(z-1)}.$$

(b):

$$\int_{|z|=10} \frac{z^9}{z^5-2} dz.$$

8. (60pts) Find the singularities of the following functions, classify them and calculate their residues:

(a)

$$\frac{1-\cos(z)}{z^3}$$

(b)

$$z^6 e^{1/z}.$$

(c)(Extra credit 10pts)

$$\frac{1}{\sin z}.$$

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9. (60pts) Calculate the following integrals using appropriate contour integrals.
(a):

$$\int_{-\infty}^{\infty} \frac{1}{x^4+4} dx.$$

(b):

$$\int_{0}^{\infty} \frac{\cos(2x)}{x^2+1} dx.$$

(c)(Extra credit 10pts):

$$\int_0^{2\pi} \frac{d\theta}{2+\sin\theta}.$$

Scratch paper

Scratch paper