

Homework assignment 7

Due date: October 20

pp.73-74

Exercise 1. Which of the following maps T from \mathbb{R}^2 into \mathbb{R}^2 are linear transformations?

(a) $T(x_1, x_2) = (1 + x_1, x_2)$;

(b) $T(x_1, x_2) = (x_2, x_1)$;

(c) $T(x_1, x_2) = (x_1^2, x_2)$;

(d) $T(x_1, x_2) = (\sin x_1, x_2)$;

(e) $T(x_1, x_2) = (x_1 - x_2, 0)$.

Exercise 3. Find the range, rank, null space, and nullity for the differentiation transformation D on the space of polynomials of degree $\leq k$:

$$D(f) = f'.$$

Do the same for the integration transformation T :

$$T(f) = \int_0^x f(t)dt.$$

Exercise 5. If

$$\alpha_1 = (1, -1), \quad \beta_1 = (1, 0)$$

$$\alpha_2 = (2, -1), \quad \beta_2 = (0, 1)$$

$$\alpha_3 = (-3, 2), \quad \beta_3 = (1, 1)$$

is there a linear transformation T from \mathbb{R}^2 into \mathbb{R}^2 such that $T\alpha_i = \beta_i$ for $i = 1, 2, 3$?

Exercise 6. Describe in coordinates (as in Exercise 1) the linear transformation T from F^2 into F^2 such that $Te_1 = (a, b)$, $Te_2 = (c, d)$ ($\{e_1, e_2\}$ is the standard basis in F^2).

Exercise 8. Describe explicitly the linear transformation from \mathbb{R}^3 into \mathbb{R}^3 that has as its range the subspace spanned by $(1, 0, -1)$ and $(1, 2, 2)$.

Exercise 10. Let V be the set of complex numbers regarded as a vector space over the field of *real* numbers (with usual operations). Find a function from V into V that is a linear transformation on V , but that is not a linear transformation on \mathbb{C}^1 , i.e., that is not complex linear.

Exercise 13. Let V be a vector space and T a linear transformation from V into V . Prove that the following two statements about V are equivalent.

(a) The intersection of the range of T and the null space of T is the zero subspace of V .

(b) If $T(T\alpha) = 0$, then $T\alpha = 0$.

Bonus exercise 12. Let V be an n -dimensional vector space over the field F and let T be a linear transformation from V into V such that the range and null spaces of T are identical. Prove that T is even. (Can you give an example of such a linear transformation T ?)