Homework assignment 3
Due date: September 22

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Exercise 1. Let

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \end{pmatrix}.$$

Compute ABC and CAB.

Exercise 3. Find two different 2×2 matrices A such that $A^2 = 0$ but $A \neq 0$.

Exercise 5. Let

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix}.$$

Is there a matrix C such that $CA = B$?

Exercise 7. Let A and B be 2×2 matrices such that $AB = I$. Prove that $BA = I$.

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Exercise 3. For each of the two matrices

$$A = \begin{pmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix}$$

use elementary row operations to discover whether it is invertible, and to find the inverse in case it is.

Exercise 4. Let

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix}.$$

For which X does there exist a scalar c such that $AX = cX$?

Exercise 8. Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Prove, using elementary row operations, that A is invertible if and only if $(ad - bc) \neq 0$.

Exercise 9. An $n \times n$ matrix is called *upper-triangular* if $A_{ij} = 0$ for $i > j$, that is, if every row below the main diagonal is 0. Prove that an upper-triangular matrix is invertible if and only if every entry on its main diagonal is different from 0.

Bonus Exercise 12. Prove that the matrix

$$\begin{pmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n+1} \\ \vdots & \vdots & & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \cdots & \frac{1}{2n-1} \end{pmatrix}$$

is invertible and A^{-1} has integer entries.