Homework assignment 10
Due date: November 10

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Exercise 2. Let $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the basis for $\mathbb{C}^3$ defined by

$$\alpha_1 = (1, 0, -1) \quad \alpha_2 = (1, 1, 1) \quad \alpha_3 = (2, 2, 0).$$

Find he dual basis of $\mathcal{B}$.

Exercise 3. If A and B are $n \times n$ matrices over the field F, show that trace(AB) =trace(BA). Now show that similar matrices have the same trace.

Exercise 4. Let V be the vector space of all polynomial functions p from $\mathbb{R}$ to $\mathbb{R}$ that have degree 2 or less:

$$p(x) = c_0 + c_1 x + c_2 x^2.$$

Define three linear functionals on V by

$$f_1(p) = \int_0^1 p(x)dx, \quad f_2(p) = \int_0^2 p(x)dx, \quad f_3(p) = \int_0^{-1} p(x)dx.$$

Show that $\{f_1, f_2, f_3\}$ is a basis for V^* by exhibiting the basis for V of which it is dual.

Exercise 8. Let W be the subspace of $\mathbb{R}^5$ that is spanned by the vectors

$$\alpha_1 = e_1 + 2e_2 + e_3, \quad \alpha_2 = e_2 + 3e_3 + 3e_4 + e_5,$$

$$\alpha_3 = e_1 + 4e_2 + 6e_3 + 4e_4 + e_5.$$

Find a basis for W^0.

Exercise 11. Let W_1 and W_2 be the subspaces of a finite-dimensional vector space V.

(a) Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.

(b) Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$.

Bonus exercise 14. Let F be a field of characteristic 0, and let V be a finite-dimensional vector space over F. If $\alpha_1, \ldots, \alpha_m$ are finitely many vectors in V, each different from the zero vector, prove that there is a linear functional f on V such that

$$f(\alpha_i) \neq 0, \quad i = 1, \ldots, m.$$