Homework assignment
Due date: September 10

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Exercise 1. Verify that the set of complex numbers of the form $x+y\sqrt{2}$, where x and y are rational, is a subfield of the field of complex numbers.

Exercise 3. Are the following two systems of linear equations equivalent? If so, express each equation in each system as a linear combination of the equations in the other system.

$$\begin{array}{rrrl} -x_1+ & x_2+ & 4x_3 & =0 \\ x_1+ & 3x_2+ & 8x_3 & =0 \\ \frac{1}{2}x_1+ & x_2+ & \frac{5}{2}x_3 & =0 \end{array} \qquad \begin{array}{rrrl} x_1 & & -x_3 & =0 \\ & x_2 & +3x_3 & =0 \end{array}$$

Exercise 6. Prove that if two homogeneous systems of linear equations in two unknowns have the same solutions, then they are equivalent.

Exercise 8. Prove that each field of characteristic zero contains a copy of the rational number field.

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Exercise 2. If

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$$

find all solutions of $AX=0$ by row-reducing A.

Exercise 5. Prove that the following two matrices are *not* row-equivalent:

$$\begin{pmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}.$$

Exercise 6. Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

be a 2×2 matrix with complex entries. Suppose that A is row-reduced and also that $a+b+c+d=0$. Prove that there exactly three such matrices.

Exercise 7. Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.