

Project 6

(due by 12/21/05 – 10:30am)

This project concerns some problems related to the Jordan form of a linear operator – you should look a little into Chapter 8, notably the section on square roots. But not many details will be necessary.

Discuss the problems below in a concise and precise essay, at most 5 typed pages long. Whenever you use a reference, quote it and do not copy. Use your own words.

1. Suppose $T: \mathbb{C}^4 \rightarrow \mathbb{C}^4$ is defined by $T(z_1, z_2, z_3, z_4) = (z_2, z_3, z_4, 0)$. Prove that T has no square root. More precisely, prove that there does not exist a linear operator $S: \mathbb{C}^4 \rightarrow \mathbb{C}^4$ such that $S^2 = T$.

2. Define $N: \mathbb{F}^5 \rightarrow \mathbb{F}^5$ by

$$N(x_1, x_2, x_3, x_4, x_5) = (2x_2, 3x_3, -x_4, 4x_5, 0).$$

Find a square root of $I + N$.

3. Prove that if V is a complex vector space, then every invertible operator on V has a cubic root.