Project 4

(due by 12/21/05 – 10:30am)

This project concerns some simple applications of Linear Algebra to Fibonacci Numbers. Discuss the problem below in a concise and precise essay at most 5 typed pages long. Whenever you use a reference, quote it and do not copy. Use your own words.

Let a_n denote the basic sequence of Fibonacci numbers defined by the recursive relation

$$a_{n+2} = a_n + a_{n+1}, \quad a_0 = 1, a_1 = 1\,.$$

Work out the problems below and find an explicit formula for a_n.

1. Consider an operator T on the vector space $\mathbb{R}^2$ such that T maps the vector (x, y) to the vector $(y, x+y)$. Show that T maps (a_{n-2}, a_{n-1}) to (a_{n-1}, a_n), where a_n is the Fibonacci sequence. Write the matrix A of T in the standard basis and prove that

$$A^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_n \\ a_{n-1} \end{bmatrix}.$$

2. Diagonalize the operator T by finding its eigenvalues and eigenvectors. Show that the eigenvectors v_1, v_2 form a basis in $\mathbb{R}^2$. If B denotes the diagonal matrix of T with respect to this basis, verify that $A = P^{-1}BP$, where the columns of the transition matrix P are v_1, v_2 in terms of the coordinates with respect to the standard basis.

3. Find B^n and conclude that $A^n = P^{-1}B^nP$ from $A = P^{-1}BP$. Now easily find A^n and write an explicit formula for the n-th Fibonacci number.