# Project 3

(due by 12/21/05 – 10:30am)

*This project concerns applications of Linear Algebra to approximating continuous functions by polynomials. Discuss the problem below in a concise and precise essay, at most 5 typed pages long. Whenever you use a reference, quote it and do not copy. Use your own words.*

We want to use the idea of orthogonal projection to approximate a function as in Chapter 6, pp111-116. Consider the function cosh defined by

$$\cosh x = \frac{e^x + e^{-x}}{2} \ .$$

Recall some basic properties of this important function (including its graph, which is also known as a *catenary*, since a homogeneous chain freely suspended beween two points will hang conforming to this shape. Note in particular that cosh is an *even* function. We now restrict attention to the fixed interval $[-2,2]$. On the real vector space $C[-2,2]$ of all continuous function $f\colon [-2,2] \to \mathbb{R}$ we work with the inner product

$$< f, g >= \int_{-2}^{2} f(x)g(x)dx \ ,$$

as usual. Let $U$ denote the subspace of all polynomial functions in $C[-2,2]$ of degree at most 5. Let $v \in C[-2,2]$ be the restriction of cosh to the interval $[-2,2]$ and $P_U v$ the othogonal projection of $v$ in $U$.

**1.** Discuss first that $x, x^3, x^5$ are all orthogonal to $v$ on $[-2,2]$. Why is no explicit computation necessary here? Exploit that odd-degree monomials are odd functions.

**2.** Why does the previous result guarantee that $P_U v$ actually lies in the subspace $W \subset U$ spanned by $1, x^2, x^4$. Now compute $P_U v = P_W v = u$ by applying the Gram-Schmidt process to $1, x^2, x^4$ and obtain an orthonormal basis $e_1, e_2, e_3$ of $W$. So this best approximation of $v$ on $[-2,2]$ by polynomials of degree at most 5 will be of the form $u(x) = a + bx^2 + cx^4$.

**3.** As in our text, plot $v$ and $u$ in one graph. To obtain reasonably good graphics you need to use Maple, Mathematica, or similar programs for plotting, which you should try even if you have never done it before. Also plot $v$ against its Taylor polynomial about 0, $p(x) = 1 + \frac{x^2}{2} + \frac{x^4}{24}$. In any case, decide either from the graphs or by any other method, which of these 2 polynomials $u, p$ is the better approximation. Of course, both of them are very good. Discuss any other observations you might have made.